Finite Element Analysis Answers

By

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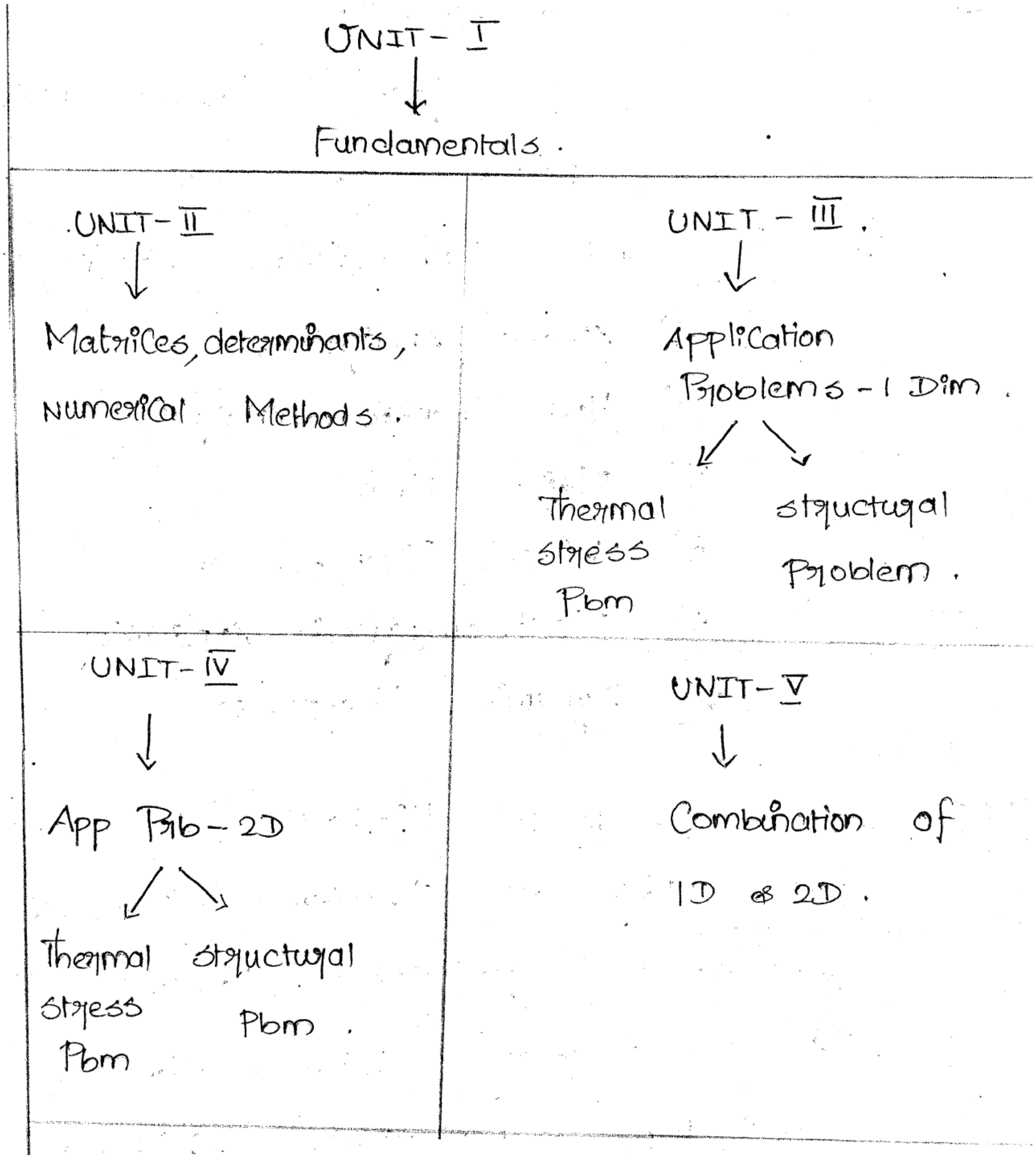
FREQUENTLY ASKED QUESTIONS IN

FINITE ELEMENT ANALYSIS



10/7/14

INTRODUCTION

* The Finite Element Method is a Num Method to solve a Problem in Engg and Mathematical Application.

i) Stress Analysis,
ii) Heat Transfer Analysis,
iii) Flued Flow.
iv) Mass Transfer and
v) Electromagnetic Application.
vi) Dynamic Response.

* These Method Can able to solve Physical Problem involving Complicated geometries, Loading and Material Property. which Cannot be solved by analytical Method.

* In these Method the domain is analysed to Carry out analysis by dividing into smaller elements.

6M (VQ) **Procedure ~~of~~ Fem (Finite Element Method):**

1. DISCRETIZATION

2. NUMBERING NOTES AND ELEMENTS

→ PRE PROCESSING

3. SELECTION DISPLACEMENT

4. DEFINING MATERIAL BEHAVIOUR.

5. DERIVATION OF ELEMENT EQUATION

6. ASSEMBLING.

7. APPLYING BOUNDARY CONDITION.

8. SOLUTION

→ ANALYSIS (PHASES)

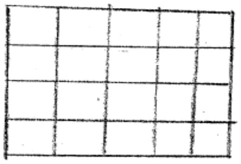
9. OBTAINING REQUIRED RESULTS FRM SOLUTION

10. CONTOUR PLOTS → POST PROCESSING

STEPS.

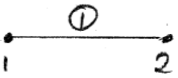
1 DESCRITIZATION :

The Body or Domain. divided into No. of Elements.

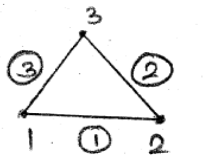


2. NUMBERING NOTES AND ELEMENTS :-

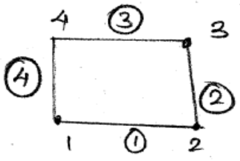
EX :-



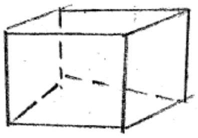
→ LINE ELEMENT.



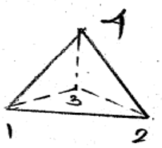
→ TRIANGULAR ELEMENT



→ QUADRILATERAL ELEMENT.



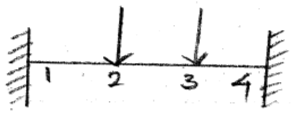
→ BRICK ELEMENT.



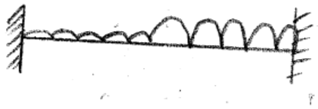
TETRA HEDRAL ELEMENT.

3. LOCATION OF NODES:-

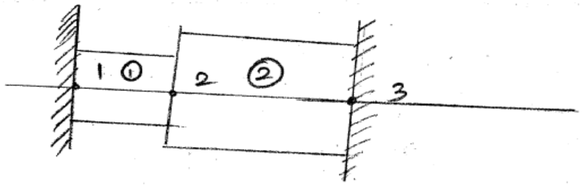
CONCENTRATED LOAD ON A BEAM.



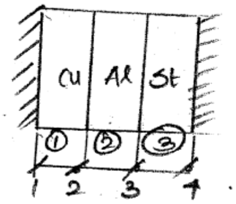
ABRUPT CHANGE IN LOAD.



CHANGE IN CROSS SECTION.

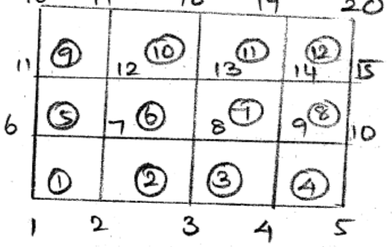


COMPOSITE VALVE WITH DIFFERENT MATERIAL.



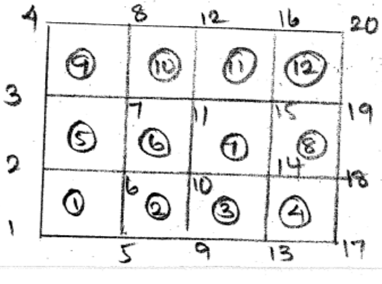
CHOICE OF NUMBERING SCHEME :

1.



→ Longer Side Numbered First.

2.



→ shorter side Numbered First

EG 1:- Consider element No : 1

Max Node NO :- 7

Min Node NO :- 1

Diff :- 6

EG 2:- consider element No :- 2

Max Node No :- 6

Min Node No :- 1

Diff :- 5 .

Hence EG :- 2 is selected because it Reduces Memory space.

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Defining the Material behaviour.

Using stress strain and strain displacement Relationship. the Results from FEA Mainly depend on Material behaviour is accurately Modelled. The M.b Model is defined by stress-strain Relationship and strain displacement relationship. Therefore,

1D → $\sigma = E \cdot \varepsilon$. → ①

E → young's Modulus

ε → strain

σ → stress.

Eqn ① is Called stress and strain Relationship,

The strain displacement is given by,

$$\varepsilon = \frac{du}{dx}.$$

Where u = displacement Field Variable along the Axial direction.

Derivation of Element Equation or Formulation of characteristics Matrix and vector,

The Element Equation is in the form of,

$$\left[[\overset{e}{k}]\right]\left[[\overset{e}{a}]\right] = \left[[\overset{e}{f}]\right]$$

Where, $[\overset{e}{k}]$ = Element stiffness Matrix,

$[\overset{e}{a}]$ = Element displacement vector,

$[\overset{e}{f}]$ = Element Force Vector.,

Assembling all the Element equation of all the elements in the descritized domain using Method of superposition, is Called direct stiffness Method, To get the global equation of the Physical domain,

$$\left[[k]\right]\left[[a]\right] = \left[[f]\right].$$

Where, [k] = Global stiffness Matrix.
[a] = Global displacement vector.
[F] = Global Force vector.

Applying the boundary condition,

The B.c's are :- classified into

i) Primary boundary condition or Essential boundary condition

secondary or Natural b.c.

) The P.b.c are applied in Global stiffness Matrix and Global Force vector.

solution,

After applying the boundary condition by Any one of the Numerical Method to solve the problem.

Obtaining Required Results From solution of displacement vector,

From s.d.v, the stress and strain values can be obtained, The strain can be obtained by,

$$\varepsilon = \frac{du}{dx} = \frac{u_2 - u_1}{x_2 - x_1}$$

u_1 & u_2 = Displacement at Node 1 & 2.

$u_2 - u_1$ = Deformation,

$x_2 - x_1$ = Change in distance or Original length of element.

By knowing the stress and strain element we can use the below Equation.

(UQ) (7) ADV / DISADV:-

* Irregular geometry can be Modelled Mor accurately and easily.

* Implementation of any type of boundary condition is possible.

* Any type of loading can be done. An isetropic Materials can be Modelled., Heterogenous Materials also.

* Elements sizes can be Varied.

* Able to solve linear and Non-linear Different loads or boundary conditions can be Modelled.

DISADVANTAGE:-

* FEA is a costlier package.

* Output is Considerably varies, depends on the Model that is element and Meshing.

* FEA softwares are not User friendly.

* Before using element of a Problem we should know about its capabilities, Nature of elements.

Applications:-

(1) structural Problem

(2) Nonstructural Problem

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17/7/14

Application of Fem.

* Structural Problem - Linear

Non-linear

* Non-structural Problem - Linear

Non-linear

(1) Linear

Stress analysis - EG:- A plate with a Hole subjected to a internal loads.

(ii) Non - linear Analysis. EG:- A Machine Element subjected to a stress More than Elastic limit.

EG:-

* Geometrical Non linearity :- A boiler shell is subjected to axial and Torsional loads.

* Eigen buckling analysis:-

Eg:- A Connecting rod is subjected to Axial Compression,

* Vibrational Analysis:-

Eg:- The beams are subjected to different types of loading.

* Types of Non - structural problems :-

i) Linear

Heat Transfer Analysis — EG:- steady state of Thermal Analysis.

↓

Linear

ii) Non linear

Thermal analysis.

on isotropic

*	Fluid Flow analysis	EG:- Fluid Flow through a Pipes or channels.
*	Electro Magnetic Analysis	EG:- Modelling ElectroMagnetic Field of Motor.

(+) (iv) Sources of Error

* The Error of the finite element solution depends on discretization by Finite Element with Meshes.

(i) Modelling Error,

It refers to diff b/n Physical system and Mathematical Model.

(ii) Discretization of Error,

It Refers to representing the degrees of freedom. Infinite degrees of freedom of a Continuous Mathematical Model.

(iii) Round of Error:-

It is Caused by Use of finite Number of bits or digits to represent the 'Real Numbers.

(IV) Inherited Error:-

It Refers to some of Previous descritization and round of Error.

(V) Manipulation Error:

It Refers to round off Error introduced by an Algorithm.

(#) U.V **VARIOUS METHODS OFOR FORMULATION OF ELEMENT PROPERTIES:-**

(i) Direct Approach:

Eg:-*) structural Problem,

*) simple Problem.

(ii) Variational Approach:-

* simple,

* sophastifated shapes.

* Variational Calculus is essential.

(#)(iii) Weighted Residual Method:-

(IV) Energy Method:

It is based on Energy balance of a system,

* stress Analysis pbm

(V) Virtual displacement Method:-

Used for stress analysis Problem.

1) A Long Rod is subjected to Loading and a Temp increase of 30°. The Total strain at a Point is Measured to be 1.2×10^{-5} & E = 200 Gpa, thermal diffusability $\alpha = 12 \times 10^{-6}$ °c.

Gi :-

$\Delta T = 30^\circ C$,

$E = 200$ Gpa, $= 200 \times 10^9$ Mpa.

$\alpha = 12 \times 10^{-6}$ °c

$\varepsilon = 1.2 \times 10^{-5}$.

W.K.T,

$$\sigma = E \cdot (\varepsilon - \varepsilon_0)$$

$$\varepsilon_0 = \alpha \cdot \Delta T = 12 \times 10^{-6} \times 30 = 3.6 \times 10^{-4}$$

$$\sigma = 200 \times 10^9 (1.2 \times 10^{-5} - 3.6 \times 10^{-4})$$

$$\sigma = -6.96 \times 10^7 \text{ mpa (comp stress)}$$

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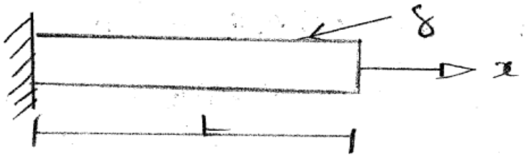
② Considered a rod shown in fig where strain at any pt of x is given by a equation,

$\varepsilon x = 1 + 2x^2$, Find the tip displacement (δ) For the given Application.

Fig :-



W.K.T strain is change in length by Original length

$$\varepsilon_x = \frac{\delta L}{L}$$

ε_x = strain along 'x'-axis

δ_L = change in length in mm by Original length in mm.

$$\delta = \varepsilon_x . L \longrightarrow ①$$

Integrate the above eqn with respect to 'x' from 0 to l.

$$\delta = L \int_0^L \varepsilon x$$

$$= L \int_0^L (1+2x^2)\,dx$$

$$= L \left[x + \frac{2x^3}{3}\right]_0^L$$

$$= L^2 \left[L + \frac{2L^3}{3}\right]$$

$$= L^2 + \frac{2L^4}{3}$$

//

③ At a Point in a stressed Material the Cartesian components of stress, Normal stress in x direction is given by "σ_x" = 70 Mpa, σ_y = 60 Mpa σ_z = 50 Mpa, τ_{xy} = 20 Mpa, τ_{yz} = −20 Mpa, τ_{xz} = 0. and $\cos\alpha = \frac{12}{25}$, $\cos\beta = \frac{15}{25}$, $\cos\gamma = \frac{16}{25}$. Find out i) Resultant stress, ii) Normal stress, (σ_n) iii) shear stress. (σ_s)

Given :-

σ_x = 70 Mpa, σ_y = 60 Mpa, σ_z = 50 Mpa

τ_{xy} = 20 Mpa, τ_{yz} = −20 Mpa, τ_{xz} = 0.

$\cos\alpha = 12/25$, $\cos\beta = 15/25$, $\cos\gamma = 16/25$.

sol :-

$$T_x = \sigma_x \cdot \cos\alpha + \tau_{xy} \cdot \cos\beta + \tau_{xz} \cos\gamma$$

$$T_y = \sigma_y \cdot \cos\beta + \tau_{xy} \cdot \cos\alpha + \tau_{yz} \cdot \cos\gamma.$$

$$T_z = \sigma_z \cos\gamma + \tau_{xz} \cos\alpha + \tau_{yz} \cos\beta.$$

Resultant stress:-

$$\sigma_r = \sqrt{T_x^2 + T_y^2 + T_z^2}$$

Normal stress:-

$$\sigma_n = T_x \cos^2\alpha + T_y \cos^2\beta + T_z \cos^2\gamma$$

Shear stress:-

$$\sigma_s = \sqrt{\sigma_r^2 - \sigma_n^2} .$$

$$T_x = 70 \times \frac{12}{25} + 20 \times \frac{15}{25} \cancel{+} = 45.6 \text{ Mpa} .$$

$$T_y = 60 \times \frac{15}{25} + 20 \times \frac{12}{25} \cancel{+} -20 \times \frac{16}{25} = 32.8 \text{ Mpa}$$

$$T_z = 50 \times \frac{16}{25} \cancel{+} -20 \times \frac{15}{25} .= 20 \text{ Mpa}$$

i) R.S:-

$$\sigma_r = \sqrt{(45.6)^2 + (32.8)^2 + (20)^2}$$

$$\boxed{= 59.625 \text{ Mpa}} .$$

ii) N.S:-

$$\sigma_n = 45.6 \times \left(\frac{12}{25}\right)^2 + 32.8 \left(\frac{15}{25}\right)^2 + 20 \left(\frac{16}{25}\right)^2$$

$$\boxed{= 30.50 \text{ Mpa}}$$

iii) S.S:-

$$\sigma_s = \sqrt{\sigma_r^2 - \sigma_n^2}$$

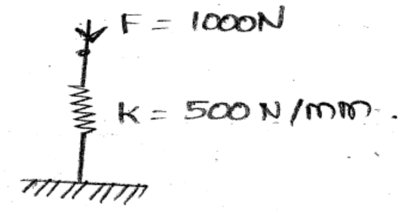
$$= \sqrt{(59.62)^2 - (30.50)^2}$$

$$\boxed{= 51.22 \text{ Mpa}} .$$

22/7/14

④ A linear elastic spring is subjected to a course of 1000N As shown in fig. Calculate the displacement and potential Energy of the spring system.

G:-



To find:-

i) x,

ii) potential energy

Sol:-

W.K.T, Total potential energy is given by,

$$\pi = U - H \longrightarrow ①$$

$U \longrightarrow$ strain Energy $\rightarrow U = \frac{1}{2}(k \cdot x) \cdot x$.

$H \longrightarrow$ Work done by external force is given by $\longrightarrow F \cdot x$.

Now sub, U & H in eqn (1),

$$\pi = \frac{1}{2}(kx^2) - F \cdot x \longrightarrow ③$$

For stationary value of, $\pi = \frac{\partial \pi}{\partial x} = 0$

$$\frac{1}{2} \times x \cdot 2x - F = 0$$

$$kx - F = 0 \longrightarrow ②$$

Now sub F & K in eqn (2)

$500x - 1000 = 0$ $\quad \boxed{x = 2\,mm}$.

$x = \frac{1000}{500}$

Now sub x in eqn ③.

$$\pi = \frac{1}{2}(K(2)^2) - F\cdot 2$$

$$= \frac{1}{2} Kx^2 - 2F = \frac{1}{2}\cdot(500)(2)^2 - 2(1000)$$

$$\boxed{\pi = -1000\ Nmm}$$

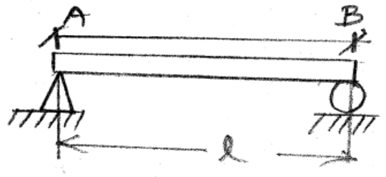
~~$= \frac{4(500)^2}{2} - 2(1000)$~~

— × —

Maximum deflection, $y_{max} = \frac{Wl^3}{48EI}$.

5. A beam of AB of span of length l simply supported at ends and carrying a concentrated load W, at Centre C as shown in the fig. Determine the deflectio at which span by Using Rayleigh's Ritz Method. Determine with the exact solution.

Gi:-



To find:-

Deflection at Mid span, y_{max}.

Solution:

Deflection $y = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l} \rightarrow ①$

The Total Potential energy of the beam is given by,

$\boxed{\pi = U - H}$ —— ②

The strain energy (U) of the beam due to Loading is given by,

$$U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 \cdot dx \longrightarrow (3)$$

From eqn W.K.T,

$U = \frac{EI}{4l^3}\pi^4\left[a_1^2 + 81a_2^2\right]$ —— ④

The Work done by external Force,

$$\boxed{H = W \cdot Y_{max}}$$

W.K.T,

$$y = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{3\pi x}{l}$$

In the span deflection is Maximum at the Middle point

$$\boxed{x = l/2}$$

Now apply $x = l/2$ in ①.

$$y = a_1 \sin\frac{\pi \cancel{l}}{2\cancel{l}} + a_2 \sin\frac{3\pi \cancel{l}}{2\cancel{l}}$$

$$y = a_1(1) + a_2(-1)$$

$$\boxed{y_{max} = a_1 - a_2}$$

sub y_{max} in H,

$$\boxed{H = W\cdot(a_1 - a_2)}.$$

Now, sub U & H in eqn

$$\pi = U - H$$

$$\pi = \frac{EI}{4l^3}\pi^4\left[a_1^2 + 81a_2^2\right] - W\cdot(a_1 - a_2).$$

For stationary value of π, the following condition must be satisfied,

$$\frac{\partial\pi}{\partial a_1} = 0, \quad \frac{\partial\pi}{\partial a_2} = 0.$$

$$\frac{EI\pi^4}{4l^3}\left[2a_1 + 0\right] - W = 0$$

$$W = \frac{EI\pi^4}{\cancel{4}_{2}l^3}\times\cancel{2}a_1$$

$$a_1 = \frac{2Wl^3}{EI\pi^4}$$

$$\frac{EI\pi^4}{4l^3}\left[0 + 162a_2\right] + W = 0$$

$$\frac{EI\pi^4\times 162a_2}{4l^3} = -W.$$

$$162a_2 = -\frac{W4l^3}{EI\pi^4}$$

$$a_2 = -\frac{W4l^3}{EI\pi^4\times 162} = \frac{-2Wl^3}{81\times EI\pi^4}.$$

W.k.T,

$$y_{max} = a_1 - a_2$$

$$= \frac{2Wl^3}{EI\pi^4} + \frac{2Wl^3}{EI\pi^4 \times 81}$$

$$\left(\frac{1}{48} = 0.02\right.$$

$$= \frac{162Wl^3 + 2Wl^3}{81EI\pi^4} = \frac{164Wl^3}{81EI\pi^4} = \frac{Wl^3}{48EI}.$$

$$\text{Clue} = \frac{2l^3W}{EI\pi^4}\left(1 + \frac{1}{81}\right).$$

—x—

⑥ (*) U.Q.

The diff eqn of a Physical phenomenon is given by $\frac{d^2y}{dx^2} + 500x^2 = 0$, which is $0 \le x \le 1$. the Trial Function $y = a_1(x - x^4)$. the boundary conditions are $y(0) = 0$, $y(1) = 0$. calculate the value of parameter a_1 by the Following Method.

Weighted Residue Method :-

1) Point Collogation Method
2) Sub domain collogation Method
3) Least - sauare collogation Method
4) Galerkin's Collogation Method.

Sol:- $\frac{d^2y}{dx^2} + 500x^2 = 0 \longrightarrow (1)$

whether the Trial Function satisfies the boundary conditions or not.

When $x=0$, $y=0$

When $x=1$, $y=0$

$y = a_1(x-x^4) = 0$, at $x = 0$ & 1

Hence it satisfies B.C.

i) Point Collogation Method:-

$$y = a_1(x-x^4).$$

$$\frac{dy}{dx} = a_1(1-4x^3).$$

$$\frac{d^2y}{dx^2} = a_1(-12x^2) = -12a_1x^2.$$

sub $\frac{d^2y}{dx^2}$ in the given above eqn (1).

$$-12a_1x^2 + 500x^2 = R.$$

In Point Collogation Method,

The Residuals are set to 0.

$$\therefore R = -12a_1x^2 + 500x^2 = 0$$

$$(-12a_1 + 500)x^2 = 0$$

$$a_1 = 41.66.$$

Hence trial function is,

$$y = 41.66(x-x^4).$$

ii) Subdomain Method:-

This Method requires $\int_0^1 R dx = 0$

$$\int_0^1 (-12a_1x^2+500x^2)\,dx = 0$$

$$\left[-12a_1 \frac{x^3}{3} + 500\frac{x^3}{3}\right]_0^1 = 0$$

$$-\frac{12a_1}{3} + \frac{500}{3} = 0$$

$$-12a_1 + 500 = 0$$

$$a_1 = 41.66$$

∴ Trial function is $y = 41.66(x-x^4)$

iii) Least square Method:-

$$I = \int_0^1 R^2 dx$$

It can also be written as

$$\frac{\partial I}{\partial a_1} = \int_0^1 R\frac{\partial R}{\partial a_1}\,dx$$

$$\frac{\partial I}{\partial a_1} = \int_0^1 [-12a_1x^2+500x^2]\frac{\partial R}{\partial a_1}\,dx$$

$$R = -12a_1x^2+500x^2.$$

$$\frac{\partial R}{\partial a_1} = -12x^2.$$

$$\frac{\partial I}{\partial a_1} = \int_0^1 [-12a_1x^2+500x^2][-12x^2]\,dx$$

$$= \int_0^1 [144a_1x^4 - 6000x^4]\,dx = 0.$$

$$\left[144\, a_1 \frac{x^5}{5} - 6000\frac{x^5}{5}\right]_0^1 = 0$$

$$\frac{144a_1}{5} - \frac{6000}{5} = 0$$

$$144a_1 - 6000 = 0$$

$$a_1 = 41.66 .$$

Trial function is $y = 41.66(x - x^4)$

iv) Galevikn's Method:-

In this Method, Trial function itself is considered as the weighting function, W_i.

$$\int_0^1 W_i R\, dx = 0$$

$$\int_0^1 a_1 (x - x^4)(-12a_1 x^2 + 500x^2)\, dx = 0$$

$$a_1 \int_0^1 (-12a_1 x^3 + 500x^3 + 12a_1 x^6 - 500x^6)\, dx = 0$$

$$a_1 \left[-12a_1 \frac{x^4}{4} + 500\frac{x^4}{4} + 12a_1\frac{x^7}{7} - 500\frac{x^7}{7}\right]_0^1 = 0$$

$$a_1 \left[-3a_1 + \frac{500}{4} + \frac{12a_1}{7} + -\frac{500}{7}\right] = 0$$

$$a_1 \left[-3a_1 + \frac{12a_1}{7} - 53.5714\right] = 0$$

$$a_1 \left[-1.2857\, a_1 - 53.5714\right] = 0$$

$$-1.2857 a_1^2 - 53.5714 a_1 = 0$$

$$a_1 = 41.66$$

The trial function is

$$y = 41.66(x - x^4).$$

7) The differential equation of a physical phenomenon is given by,

$$\frac{d^2y}{dx^2} + 50 = 0 ; \quad 0 \leq x \leq 1 .$$

The Trial function $y = a_1 x (10-x)$

The boundary conditions are $y(0)=0$; $y(1)=0$, calculate the value of a?

S:-

i)
$$y = a_1 x (10-x) .$$
$$\frac{dy}{dx} = 10a_1 - 2a_1 x .$$
$$\frac{d^2y}{dx^2} = -2a_1 .$$

then, $-2a_1 + 50 = 0 \Rightarrow a_1 = 25 .$

Hence, the Trial function is,

$y = 25x (10-x) .$

ii) Subdomain Method:-

This Method requires $\int_0^1 R dx = 0$

$$\int_0^1 [-2a_1 + 50] dx = 0$$
$$[-2a_1 x + 50x]_0^1 = 0$$
$$-2a_1 + 50 = 0 , \ a_1 = 25 .$$

Hence the Trial function, $y = 25x(10-x)$

iii) Least square Method:-

This Method requires, $I = \int_0^1 R^2 dx$

It can also be written as,

$$\frac{\partial I}{\partial a_1} = \int_0^1 R \frac{\partial R}{\partial a_1} dx \longrightarrow ② .$$

W.K.T, $R = -2a_1 + 50$

$$\frac{\partial R}{\partial a_1} = -2$$

$$\therefore \frac{\partial I}{\partial a_1} = \int_0^1 [-2a_1 + 50]\ [-2]\, dx = 0$$

$$\Rightarrow [4a_1x_1 - 100x]_0^1 = 0$$

$$\Rightarrow 4a_1 - 100 = 0 \ , \ a_1 = 25 .$$

Hence the Trial function is, $y = 25x(10-x)$.

iv) Galerkin's Method :- In this Method Trial function itself is considered as the weighting function, w_i

$$\int_0^1 w_i\, R\, dx = 0 .$$

$$\int_0^1 a_1 x(10-x)\ (-2a_1 + 50)\, dx = 0 .$$

$$a_1 \int_0^1 (10x - x^2)\ (-2a_1 + 50) dx = 0$$

$$a_1 \int_0^1 (-20a_1 x + 2a_1 x^2 + 500x - 50x^2)\, dx = 0 .$$

$$a_1 \left[-20a_1 \frac{x^2}{2} + 2a_1 \frac{x^3}{3} + 500\frac{x^2}{2} - 50\frac{x^3}{3}\right]_0^1 = 0 .$$

$$a_1 \left[-10a_1 + \frac{2a_1}{3} + 250 - \frac{50}{3}\right] = 0$$

$$a_1 \left[-10a_1 + \frac{2a_1}{3} + 233.33\right] = 0$$

$$a_1 \left[-9.333a_1 + 233.33\right] = 0$$

$$-9.333a_1^2 + 233.33a_1 = 0 .$$

31/7/14. (U·V)(+)

*Hook's law:-

$$E = \frac{\sigma}{\varepsilon}$$

For a general an isotropic Material the components of stress or expressed as linear Model of six components of strain & Vice Versa. is Called as Generalised Hook's law.

Where,

D → Material stiffness Matrix, while the inverse is called as

D^{-1} Material Flexibility. Matrix

The above said Turn is called as Hook's law for a linear Elastic Isotropic & Homogeneous Material.

*Temperature effects:-

For Isotropic Material the Temp rise (ΔT) Results in a uniform strain, which depends on the coefficient of linear expansion (α) of the Material, where,

α → change in length per Unit. Temperature rise is assumed to be a constant within the range of Variation of temperature and also the strain does not cause any stresses

* when the body is Free to deform.

* The initial strain,

$$\varepsilon_0 = [\alpha . \Delta T , \alpha . \Delta T , \alpha \Delta T, 0, 0, 0].$$

* For plain strain & stress,

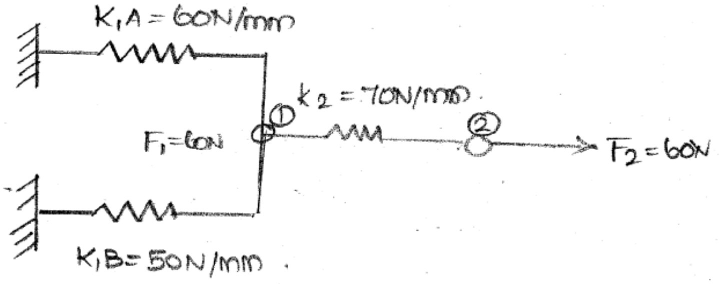
$$\sigma = [\sigma_x . \sigma_y . \tau_{xy}].$$

$$\varepsilon = [\varepsilon_x . \varepsilon_y . \gamma_{xy}].$$

— x —

(U.Q)

8) Find the potential Energy for the given spring system,



G:-

$K_1A = 60$ N/mm.

$K_1B = 50$ N/mm

$F_1 = 60$N

$F_2 = 60$N

S:-

The Total potential Energy,

$$TPE = \Pi = \frac{1}{2} K A \delta_{1A}^2 + \frac{1}{2} K_{1B} \delta_{1B}^2 + \frac{1}{2} k_2 \delta_2^2 - F_1 a_1 - F_2 a_2 .$$

Where, $\delta_{1A} = a_1$, $\delta_{1B} = a_1$,

$\delta_2 = (a_2 - a_1)$.

$$\therefore \; T_{PE} = \pi = \frac{1}{2} k_{1A} a_1^2 + \frac{1}{2} k_{1B} a_1^2 + \frac{1}{2} k_2 (a_2 - a_1)^2 - F_1 a_1 - F_2 . a_2$$

For Equilibrium 2 degrees of Freedom, we need to Minimise π. With respect to a_1, a_2.

$$\therefore \; \frac{\delta \pi}{\partial a_1} = k_{1A} a_1 + k_{1B} a_1 - k_2 (a_2 - a_1) - F_1 \quad \hookrightarrow ①.$$

$$\therefore \; \frac{\partial \pi}{\partial a_2} = k_2 (a_2 - a_1) - F_2 = 0 \longrightarrow ②.$$

From eqn ②,

$$k_2 (a_2 - a_1) - F_2 = 0$$

$$70 (a_2 - a_1) - 60 = 0$$

$$a_2 - a_1 = \frac{60}{70} = 0.857 \text{ in } ①$$

$$= 60 a_1 + 50 a_1 - 70 (0.857) - 60 = 0$$

$$= 110 a_1 - 119.99 = 0$$

$$\frac{119.99}{110} = a_1$$

$\boxed{a_1 = 1.09}$, $\boxed{a_2 = 1.947}$

UNIT-I	CHAPTER	THEORY	DERIVATION	PBM
*	FEM	✓		
*	Source of Error	✓		
*	Application	✓		
*	ADV / DISADVANTAGES	✓		
*	Number & Node	✓		
*	Types & Elements	✓		
*	Variational Method	✓		✓
*	Galarikan Method	✓		✓
*	Saint V. principle	✓		
*	Total Potential Energy.		✓	✓
	Temperature effect	✓		
	Hooks law	✓		
	Stress strain Problem	✓		✓

(÷) . UNIT - II .

* Matrices / Determinants .

* Sky line Matrix .

* Numerical Method .

— × — .

(UQ)

PROPERTIES OF MATRICES & DETERMINANTS

(i) Row & column vector →

Eg :- $d = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

Eg :- $d = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(ii) Addition and subtraction,

Eg :- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

Eg :- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$.

(iii) Multiplication,

Eg :- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$$A \times B = \begin{bmatrix} 3+2 & 2+8 \\ 9+4 & 6+16 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 13 & 22 \end{bmatrix} .$$

iv) Multiplication by scalar :-

$$A = \begin{bmatrix} 10000 & 4500 \\ 4500 & -6000 \end{bmatrix}$$

$$A = 10^3 \begin{bmatrix} 10 & 4.5 \\ 4.5 & -6 \end{bmatrix}$$

v) Transposition,

Eg:- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

vi) Square Matrix,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$$

vii) Diagonal Matrix,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

viii) Identity Matrix,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ix) Symmetrics Matrix,

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -6 & -2 \\ 0 & -2 & 6 \end{bmatrix}$$

x) Upper Triangular Matrix :-

$$A = \begin{bmatrix} 2 & 1 & -6 \\ 0 & 5 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

xi) Lower Triangular Matrix :-

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & -4 \end{bmatrix}$$

xii) Determinant of Matrix :-

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}(a_{22} * a_{33} - a_{23} \cdot a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}).$$

— x —

10/9/14

Q1. G. that area of a Δ with corners at $(x_1\ y_1)$, $(x_2\ y_2)$, (x_3, y_3) can be written in the form of $A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$. Determine the area of Δ with corners at (1,1), (4,2) & (2,4).

S:- $A = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 4 \end{vmatrix}$

$$= \frac{1}{2}\{1(16-4) - 1(4-2) + 1(2-4)\}$$

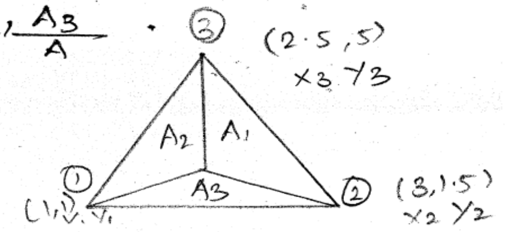
$$= \frac{1}{2}\{12-2-2\} = \frac{1}{2}\{12-4\} = \frac{1}{2} \times 8$$

$$= 4.$$

— x —

8 Marks

Q2. For the Δ of the fig. given below the interior point P at (2,2) divides 3 areas namely A_1, A_2, A_3. Determine $\frac{A_1}{A}$, $\frac{A_2}{A}$, $\frac{A_3}{A}$.



$$A = \frac{1}{2}\begin{vmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ 1 & X_3 & Y_3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 1.5 \\ 1 & 2.5 & 5 \end{vmatrix}$$

$$= \frac{1}{2}\left\{1(15-3.75) - 1(5-2.5) + 1(1.5-3)\right\}$$

$$= \frac{1}{2}(11.25 - 2.5 - 1.5) = 3.625 \text{ units}.$$

$$A_1 = \frac{1}{2}\begin{vmatrix} 1 & X_1' & Y_1' \\ 1 & X_2 & Y_2 \\ 1 & X_3 & Y_3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 2 \\ 1 & 3 & 1.5 \\ 1 & 2.5 & 5 \end{vmatrix}$$

$$= \frac{1}{2}\left\{1(15-3.75) - 1(10-5) + 1(3-6)\right\}$$

$$= \frac{1}{2}(3.25) = 1.625 \text{ units}.$$

$$A_2 = \frac{1}{2}\begin{vmatrix} 1 & X_1 & Y_1 \\ 1 & X_2' & Y_2' \\ 1 & X_3 & Y_3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2.5 & 5 \end{vmatrix}$$

$$= \frac{1}{2}\left\{1(10-5) - 1(5-2.5) + 1(2-2)\right\}$$

$$= \frac{1}{2}(5-2.5) = 1.25 \text{ units}.$$

$$A_3 = \frac{1}{2}\begin{vmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ 1 & X_3' & Y_3' \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 1.5 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\left\{1(66-3) - 1(2-2) + 1(1.5-3)\right\} = \frac{1}{2}(3-1.5)$$

$$= \frac{1.5}{2} = 0.75 \text{ units}.$$

$$A_1/A = \frac{1.625}{3.625} = 0.44828 = N_1, \quad \frac{A_2}{A} = \frac{1.25}{3.625} = 0.34483 = N_2$$

$$A_3/A = \frac{0.75}{3.625} = 0.20689 = N_3$$

—x—

③ Given that $A = \begin{bmatrix} 8 & -2 & 0 \\ -2 & 9 & -3 \\ 0 & -3 & 3 \end{bmatrix}$, $d = \begin{Bmatrix} 2 \\ -1 \\ 3 \end{Bmatrix}$

Find ① determinant of A, ② $I - d.d^T$.

S:-

$$A = \begin{bmatrix} 8 & -2 & 0 \\ -2 & 9 & -3 \\ 0 & -3 & 3 \end{bmatrix}$$

$$= \left\{8(27-9) - (-2)(-6+0) + 0(6-0)\right\}$$

$$= 8(18) + 2(-6) = 132.$$

(2) $I = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ $d = \begin{Bmatrix} 2 \\ -1 \\ 3 \end{Bmatrix}$, $d' = \{2 \ -1 \ 3\}$

$$I - dd^T, = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{Bmatrix} 2 \\ -1 \\ 3 \end{Bmatrix} \{2 \ -1 \ 3\}$$

$$= \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Bmatrix} - \begin{Bmatrix} 4 & -2 & 6 \\ -2 & 1 & -3 \\ 6 & -3 & 9 \end{Bmatrix} = \begin{Bmatrix} -3 & 2 & -6 \\ 2 & 0 & 3 \\ -6 & 3 & -8 \end{Bmatrix}$$

$$= -3(0-9) - 2(-16+18) - 6(6+0) = -3\times-9-2\times2-6\times6$$

$$= 27 - 4 - 36 = -13$$ Ans 12.

— x —

(4) Using co-factor Method determine inverse of Matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

S:- $2(4-1) - 1(2-1) + 1(1-2) = 6 - 1 - 1 = 4$ units

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}$$

$A_1 = \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix}$, $A_2 = \begin{bmatrix} b_1 & c_1 \\ b_3 & c_3 \end{bmatrix}$, $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$= 4-1 = 3$, $= 2-1 = 1$.

$A_3 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = 1-2 = -1$,

$B_1 = [2-1] = 1$, $B_2 = [4-1] = 3$ $B_3 = [2-1] = 1$

$C_1 = -1$, $C_2 = 1$, $C_3 = [4-1] = 3$.

$$\therefore \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} \quad A^{-1} = \frac{adj \cdot A}{Det \cdot A} = \frac{\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}}{4}$$

$$= \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \frac{1}{4} \{3(9-1) - 1(3+1) - 1(1+3)\}$$

$$= 1/4 \{3\times8 - 1\times4 - 1\times4\} = 1/4 \ (24-8) = 4 \text{ units}$$

— x —

5)(*) Find the Matrix of Quadratic form for the following $x^2 - 2y^2 + 3z^2 - 4yz + 6xz$.

S:

$$[x \ y \ z] \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

$$= (a_{11}x + a_{21}y + a_{31}z)x + (a_{12}x + y a_{22} + a_{32}z)y + (a_{13}x + a_{23}y + a_{33}z)z$$

$$= a_{11}x^2 + a_{21}yx + a_{31}zx + a_{12}xy + y^2 a_{22} + a_{32}zy + a_{13}xz + a_{23}yz + a_{33}z^2$$

$$= a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + a_{21}xy + a_{12}xy + a_{32}yz + a_{23}yz + a_{31}zx + a_{13}xz$$

$$a_{21} = a_{12}.$$

$$= a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{31}xz + 2a_{23}yz$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & -2 & -2 \\ 3 & -2 & 3 \end{bmatrix} \text{Ans}$$

—— x ——

⑥ Express $Q = x_1 - 6x_2 + 3x_1^2 + 5x_1 \cdot x_2$ in the Matrix form $\frac{1}{2} \cdot x^T \cdot Qx + C^T x$.

S: W.K.T, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x^T = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$

$C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, $C^T = \begin{bmatrix} c_1 . & c_2 \end{bmatrix}$.

$$\frac{1}{2} x^T \cdot Qx + C^T \cdot x$$

$$= \frac{1}{2} [x_1, x_2] \times [a_{11}\ a_{21} + a_{12}\ a_{22}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [c_1\ c_2] \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

$$= \frac{1}{2} \left[(x_1 a_{11} + x_2 a_{21}) x_1 + (x_1 a_{12} + x_2 a_{22}) x_2 \right] + (c_1 x + c_2 x_2) \Big]$$

$$= \frac{1}{2} \left[x_1^2 a_{11} + x_2 x_1 a_{21} + x_1 x_2 a_{12} + x_2^2 a_{22} \right] + \left[c_1 x_1 + c_2 x_2 \right].$$

$$= \frac{1}{2} \left[x_1^2 a_{11} + x_2^2 a_{22} + x_1 x_2 a_{21} + x_1 x_2 a_{12} \right]$$

$$+ \left[c_1 x_1 + c_2 x_2 \right].$$

$$= \frac{1}{2} \left[x_1^2 a_{11} + x_2^2 a_{22} + 2 x_1 x_2 a_{12} \right].$$

$$+ \left[c_1 x_1 + c_2 x_2 \right]$$

$$= \begin{bmatrix} 3/2 & 5 \\ 5 & 0 \end{bmatrix}$$

$c_1 = 1$, $c_2 = -6$.

$a_{11} = 3/2$, $a_{21} = 5$.

$a_{12} = 5$, $a_{22} = 0$.

— x — .

11/9/14.

CHOLESKY FACTORISATION .

$6x_1 - x_2 - x_3 = 11.33$.

$-x_1 + 6x_2 - x_3 = 32$.

$-x_1 - x_2 + 6x_3 = 42$.

solve the above system of equation by Using Cholesky Factorisation.

S:-

Let , $A = L \cdot L^T$

$$\therefore A = \begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{33} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$= \begin{bmatrix} l_{11}^2 & l_{11}\,l_{21} & l_{11}\,l_{31} \\ l_{21}\,l_{11} & l_{21}^2 + l_{22}^2 & l_{21}l_{31} + l_{22}l_{32} \\ l_{31}\,l_{11} & l_{31}l_{21} + l_{32}l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

$l_{11}^2 = 6$, $l_{11} = \sqrt{6} = 2.449$.

$l_{11}\;l_{21} = -1$

$l_{21} = -1/2.449 = -0.4083$

$l_{11}\;l_{31} = -1$

$l_{31} = -1/2.449 = -0.4083$

$l_{21}^2 + l_{22}^2 = 6$

$l_{22}^2 = 6 - (-0.4083)^2$

$l_{22}^2 = 5.833$

$l_{22} = 2.415$

$l_{21}\, l_{31} + l_{32}\, l_{22} = -1$

$$l_{32} = \frac{-1 - l_{21}\, l_{31}}{l_{22}}$$

$$= \frac{-1 - (-0.4083)(-0.4083)}{2.415}$$

$= -0.483$

$l_{31}^2 + l_{32}^2 + l_{33}^2 = 6$

$l_{33} = \sqrt{6 - (-0.4083) - (-0.483)^2}$

$= 2.366.$

W.K.T, $L \cdot Y = B$

$$\begin{bmatrix} 2.44 & 0 & 0 \\ -0.4 & 2.415 & 0 \\ -0.4 & -0.48 & 2.366 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 11.33 \\ 32 \\ 42 \end{bmatrix}$$

$= 2.44 \times y_1 = 11.33$
$+ 0 \times y_2 + 0 \times y_3$

$y_1 = 4.643.$

$-0.4 \times y_1 + 2.415\, y_2 + 0 = 32$

$-0.4 \times 4.648 + 2.415\, y_2 = 32$

$y_2 = 14.019.$

$$-0.4Y_1 - 0.48Y_2 + 2.366Y_3 = 42$$

$$-0.4 \times 4.643 - 0.48 \times 14.019 + 2.366Y_3 = 42$$

$$Y_3 = 21.38 .$$

$$LT . X = Y \Rightarrow \begin{bmatrix} 2.44 & -0.4 & -0.4 \\ 0 & 2.415 & -0.48 \\ 0 & 0 & 2.366 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} Y_1 = 4.643 \\ Y_2 = 14.032 \\ Y_3 = 21.38 \end{bmatrix}$$

$$0x_1 + 0x_2 + 2.366x_3 = 21.38$$

$$x_3 = 9.036$$

$$x_2 . 2.415 - 0.48 \times x_3 = 14.032$$

$$x_2 = 7.6$$

$$2.44x_1 - 0.4x_2 - 0.4x_3 = 4.643$$

$$x_1 = 4.63 .$$

—— x ——

SKYLINE BANDED MATRIX :-

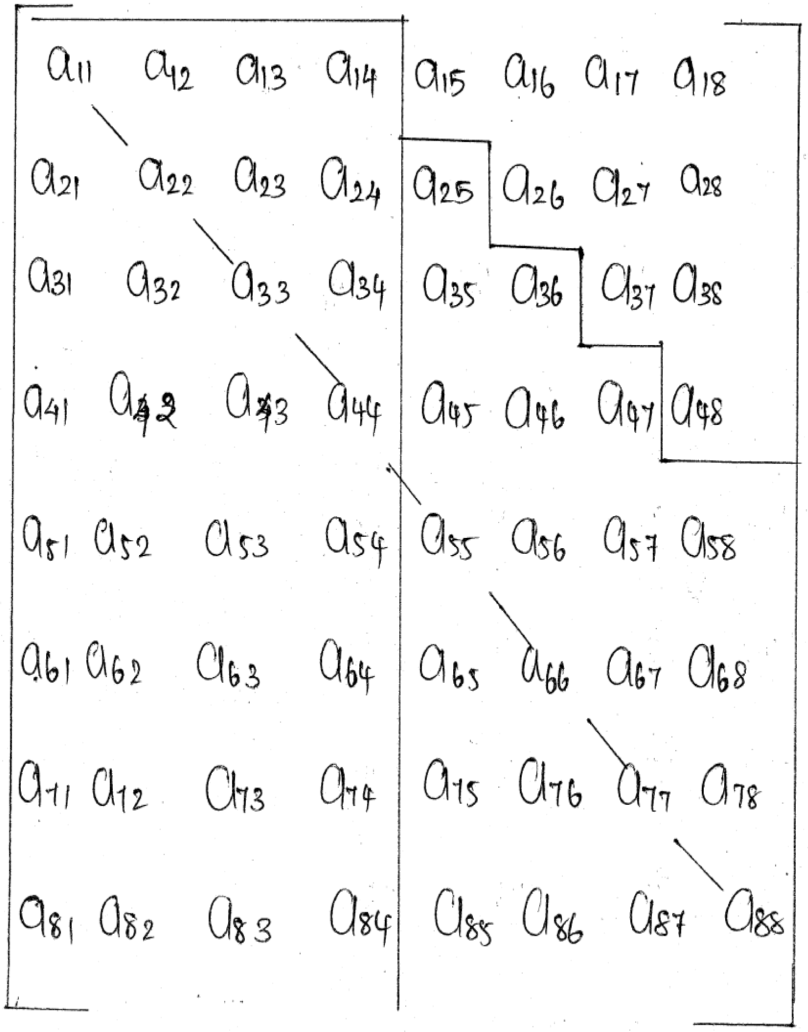
For a symmetry 8 by 8 Matrix With all Non-zero elements Determine the No. of locations Needed for Banded and skyline storage Method :-

skyline

Banded Matrix ,

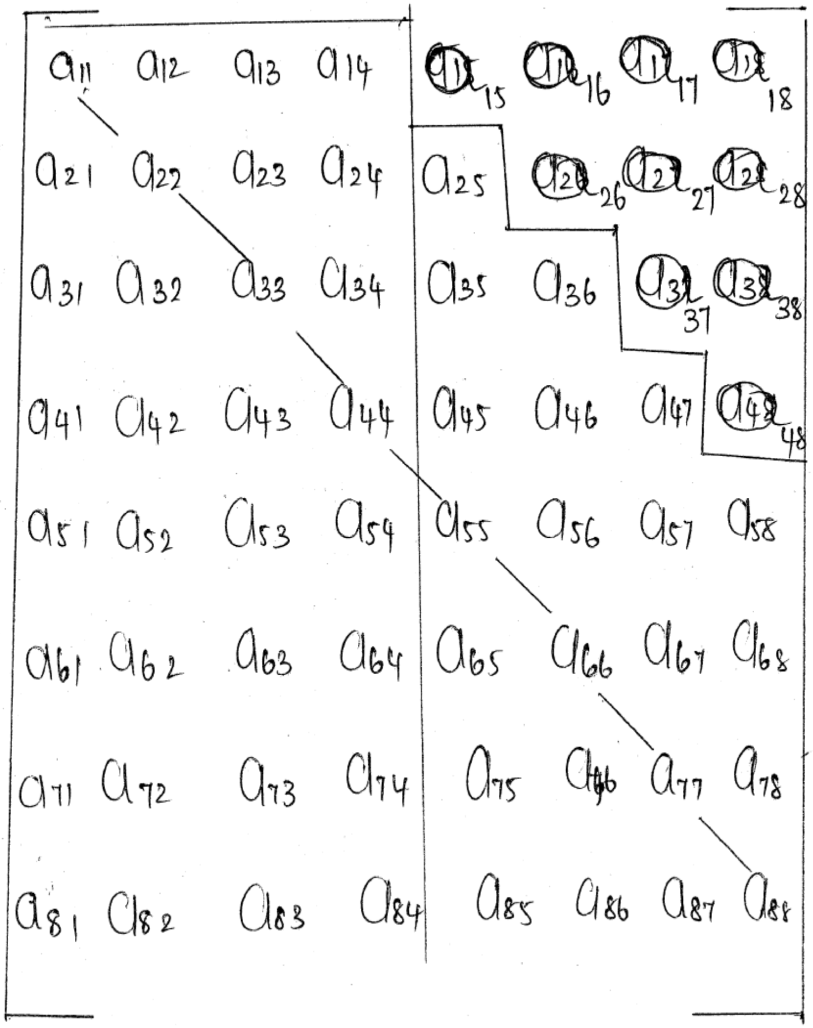
The 8×8 Matrix is given below :-

STEP 1:-



$$\left[\begin{array}{cccc|cccc}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88}
\end{array}\right]$$

STEP 2:-



$$\left[\begin{array}{cccc|cccc}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88}
\end{array}\right]$$

STEP III :-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88} \end{bmatrix}$$

STEP IV :

$$A = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{22} \\ a_{13} \\ a_{23} \\ a_{33} \\ a_{14} \\ a_{24} \\ a_{34} \\ a_{44} \\ a_{25} \\ a_{35} \\ a_{45} \\ a_{55} \end{bmatrix} \quad \begin{bmatrix} a_{36} \\ a_{46} \\ a_{56} \\ a_{66} \\ a_{47} \\ a_{57} \\ a_{67} \\ a_{77} \\ a_{58} \\ a_{68} \\ a_{78} \\ a_{88} \end{bmatrix} \qquad ID = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16. \end{bmatrix} \quad \begin{bmatrix} 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 25 \\ 26 \end{bmatrix}$$

Summary :-	D	P	T .
Prop Mat /Det		✓	✓
Cholesky		✓	✓
Skyline		✓	✓ .
Numerical Methods .		✓	✓ .
Conjugate Gradient Method			✓ .

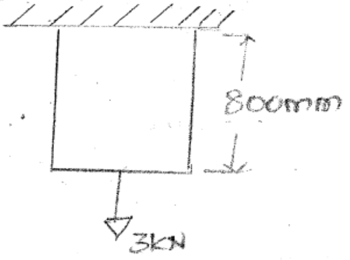
—— × —— .

UNIT-III . SUMMARY:

	D	P	T
Derivation for [k]	✓		
Derivation for Temp effect .	✓		
Structural problem		✓	
Temperature problem		✓	
Trusses		✓ .	

UNIT - III .

1. A Steel bar of Length 800mm is subjected to Axial load of 3 kN is shown in Fig. Find Elongation of bar Neglecting the self weight.



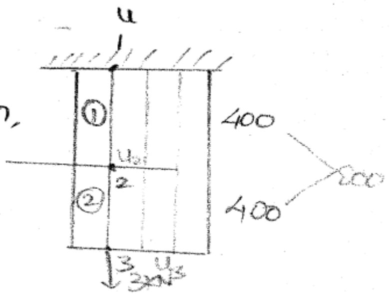
G:- Take, $E = 2\times10^5$ N/mm^2.

$A = 300$ mm^2.

To find:-

Elongation $u = ?$

S:- We can divide bar into two elements,



For One dimensional pbm, 2 Noded or 2 bar element, the Finite element equation is,

$$\frac{AE}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\{u\} = \{F\}.$$

Now, consider For element no. 1,

$$\frac{A_1E_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}.$$

Ele ② :-

$$\frac{A_2 \; E_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}.$$

For Ele ① :-

$$\Rightarrow \frac{300 \times 2\times10^5}{400} \begin{bmatrix} 1 & -1 \\ -1 & +1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}.$$

$$\Rightarrow 150\times10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

For Ele ② :-

$$\Rightarrow 150\times10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}.$$

Step iii

Assemble the Element stiffness eqn (1) and (2), We get,

$$\Rightarrow 150\times10^3 \begin{bmatrix} 1 & -1 & \\ -1 & 1+1 & -1 \\ & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow 150\times10^3 \begin{bmatrix} \cancel{1} & \cancel{-1} & \cancel{0} \\ \cancel{-1} & 2 & -1 \\ \cancel{0} & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} \cancel{F_1} \\ \cancel{F_2}\,0 \\ F_3 \end{Bmatrix}.$$

Step iv Now Applying B.C,

Displacement at Node (1) is $u_1 = 0$.

The Node, F_3 = Node ③ Neglecting the self weight;

$$2u_2 - \cancel{u_3} = 0$$

$$-u_2 + \cancel{u_3} = 3000$$

$$u_2 = \frac{\cancel{3000}^{20\times10^{-5}}}{\cancel{150}\times10^{3}}$$

$$F_3 = 3\times10^{3} N$$

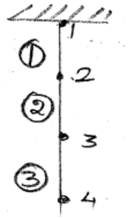
$$\therefore F_1 = F_2 = 0$$

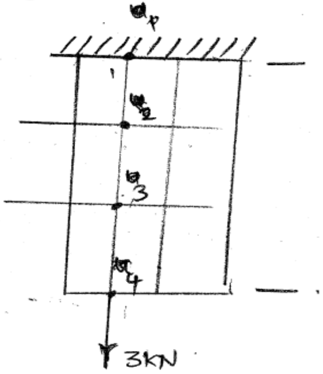
$$\therefore u_2 = 0.02 \quad , \quad u_3 = 0.04.$$

— x —

17/9/14

2. A steel bar of length of 800mm subjected to Axial load of 3kN is shown in the figure. Find the Elongation of bar and Neglecting self weight, consider three elements for the Problem.





Step 1 :- FEA Model,

Step 2 :- Element stiffness Matrix for Element No. 1.

Ele ① :-

$$\frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}.$$

Ele ② :-

$$\frac{A_2 E_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}.$$

17/9/14 Ele ③ :-

$$\frac{A_3 E_3}{l_3}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_3\\ u_4\end{Bmatrix}=\begin{Bmatrix}F_3\\ F_4\end{Bmatrix}.$$

Ele ① :-

$$=\frac{300\times 2\times 10^5}{266.66}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix}=\begin{Bmatrix}F_1\\ F_2\end{Bmatrix}.$$

$$=225.005\times 10^3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix}=\begin{Bmatrix}F_1\\ F_2\end{Bmatrix}.$$

Ele ② :-

$$=225.005\times 10^3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix}=\begin{Bmatrix}F_2\\ F_3\end{Bmatrix}.$$

$$=225.005\times 10^3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_3\\ u_4\end{Bmatrix}=\begin{Bmatrix}F_3\\ F_4\end{Bmatrix}.$$

Step III :-

Assemble the Element stiffness eqn (1), (2), (3) we get,

$$225.005\times 10^3\begin{bmatrix}1 & -1 & & \\ -1 & 1+1 & -1 & \\ & -1 & 1+1 & -1\\ & & -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{Bmatrix}=\begin{Bmatrix}F_1\\ F_2\\ F_3\\ F_4\end{Bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 = 3KN \end{Bmatrix}.$$

Neglecting self weight,

$$F_2 = 0, \quad F_3 = 0 .$$

Load is Acting at F_4 so, $F_1 = 0$,

$$\boxed{F_4 = 3 \times 10^3 N.} \quad u_1 = 0$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \\ F_4 \end{Bmatrix}.$$

$$2u_2 - u_3 = 0$$

$$-u_2 + 2u_3 - u_4 = 0$$

$$-u_3 + u_4 = 3.$$

$$u_1 = 0, \quad u_2 = 0.0133 \quad u_3 = 0.\cdot 0266$$

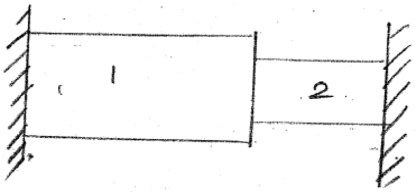
$$u_4 = 0\cdot 039$$

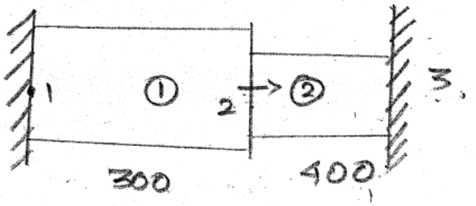
Verification:-

$$\delta_L = Pl/AE = \frac{3\times10^3\times800}{300\times2\times10^5} = 0.04.$$

3. Consider a bar as shown in fig. Axial load 200 KN at point P= & Take $A_1 = 2400mm^2$, $E_1 = 70\times10^9 N/m^2$, $A_2 = 600mm^2$, $E_2 = 200\times10^9 N/m^2$. Calculate, i) the Nodal displacement at point P, ii) stress in each Material, iii) Reaction Force.. $1m = 10^3 mm$

Step 1:-





Step 2:-

Element stiffness Matrix for

Element (1):-

$$\frac{A_1E_1}{l_1}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}=\begin{Bmatrix}F_1\\F_2\end{Bmatrix}.$$

$$2400\times70\times10^3/300\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}=\begin{Bmatrix}F_1\\F_2\end{Bmatrix}$$

$$5.6 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Ele (2) :-

$$\frac{A_2 \; E_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \frac{600 \times 200 \times 10^3}{400} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}.$$

$$\Rightarrow 3 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}.$$

$$\Rightarrow 1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 \\ -5.6 & 5.6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}.$$

$$\Rightarrow \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}.$$

Global Stiffness Matrix :-

$$\Rightarrow 1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 5.6+3 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1=0 \\ F_2 \\ F_3=0 \end{Bmatrix}.$$

$u_1 = u_3 = 0 \qquad u_2 = 0.2325$ mm

$F_1 = F_3 = 0.$

Stress ① :

$$\sigma_1 = E_1 \times \frac{(u_2 - u_1)}{l_1}$$

$$= 70 \times 10^3 \times \frac{(0.2325 - 0)}{300}$$

$$= 54.22 \; N/mm^2$$

$$1\times10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$1\times10^5 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 8.6 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$1\times10^5 \times 8.6 u_2 = 200\times10^3$$

$$\boxed{u_2 = 0.2325}$$

Stress ② :-

$$\sigma_2 = E_2 \times (u_3 - u_2) / l_2$$

$$= 200\times10^3 \; (0 - 0.2325) / 400$$

$$= -116.25$$

$$[K]\{u^*\} - \{F\} = \{R\}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0.2325 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 200\times10^3 \\ 0 \end{bmatrix}$$

$$R_1 = -1.3205\times10^5$$

$$R_2 = 0$$

$$R_3 = -0.6975\times10^5$$

24|9|17

4) A steel plate of uniform thickness 25mm is subjected to a point load of 420N as shown in fig. The plate is also subjected to self weight. If the Young's Modulus = 2×10^5 N/mm^2 and unit weight density $\rho = 0.8\times10^{-4}$ N/mm^3 calculate,

Gi:-

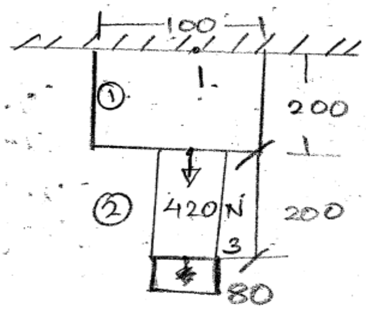
$E = 2\times10^5$ N/mm^2

$\rho = 0.8\times10^{-4}$ N/mm^3

$A_1 = 100\times25$

$A_2 = 80\times25$

$E_1 = E_2$



To find :- $u = ?$ $\sigma = ?$ $F = ?$ $K = ?$

2/2

Solution,

A steel plate is subjected to self weight Hence we have to find the body force acting at 1,2,3.

W.K.T,

Body Force vector $\{f\} = \frac{\rho A l}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$.

$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{\rho A_1 l_1}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$.

$\begin{Bmatrix}F_2\\F_3\end{Bmatrix} = \frac{\rho_2 A_2 l_2}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$.

For ele (1) :-

$\{f\} = \frac{0.8\times10^{-4}\times2000\times250}{2} = 20 = \begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \begin{Bmatrix}20\\20\end{Bmatrix}$

For ele (2) :- $= 0.8\times10^{-4}\times2000\times200/2 = 16 = \begin{Bmatrix}F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}16\\16\end{Bmatrix}$

Assemble global force vector:-

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 20+16 \\ 16 \end{Bmatrix}$$

$$= \begin{Bmatrix} 20 \\ 36+420=456 \\ 16 \end{Bmatrix}.$$

Step 3:- Stiffness Matrix for element (1).

$$\Rightarrow \frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}.$$

$$\Rightarrow \frac{2500 \times 2 \times 10^5}{200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 456 \end{Bmatrix}.$$

$$\Rightarrow 25 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 456 \end{Bmatrix}.$$

For element (2).

$$\Rightarrow \frac{2000 \times 2 \times 10^5}{200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 456 \\ 16 \end{Bmatrix}.$$

$$\Rightarrow 20 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 456 \\ 16 \end{Bmatrix}.$$

Global stiffness Matrix:-

$$\Rightarrow 1 \times 10^5 \begin{bmatrix} 25 & 25 & 0 \\ 25 & 25+20 & 20 \\ 0 & 20 & 20 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 456 \\ 16 \end{Bmatrix}.$$

$\therefore\ u_1 = 0,$

On solving,

$$45u_2 + 20u_3 = 456.$$

$$20u_2 + 20u_3 = 16.$$

$$u_2 = 1.888 \times 10^{-4}\ \text{mm}.$$

$$u_3 = 1.968 \times 10^{-4}\ \text{mm}.$$

Stress, ①:-

$$\sigma_1 = E_1 \times \frac{(u_2 - u_1)}{l_1}$$

$$= \frac{2 \cdot 0 \times 10^{5} \times (1.888 \times 10^{-4} - 0)}{200}$$

$\sigma_1 = 0.188\ \text{N/mm}^2$

$$\sigma_1 = 0.188\ \text{N/mm}^2.$$

Stress ②:-

$\sigma_2 = 0.0088\ \text{N/mm}^2$

$$\sigma_2 = E_2 \times (u_3 - u_2)/l_2.$$

$$= 2 \times 10^5 \times (1.968 \times 10^{-4} - 1.888 \times 10^{-4})$$

$$\sigma_2 = 0.0088\ \text{N/mm}^2.$$

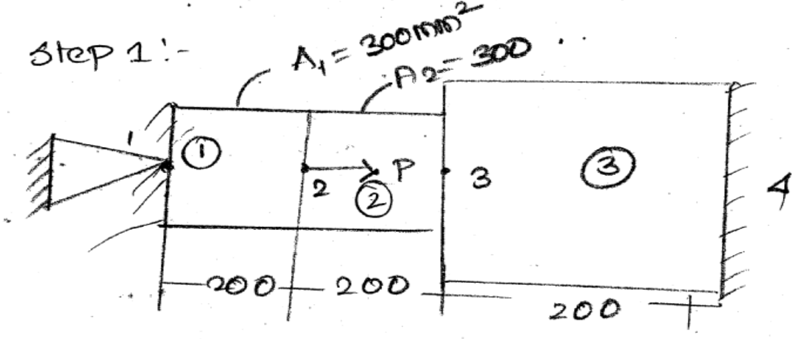
5) Consider a bar as shown in fig.
Calculate the following,

i) Nodal displacement,
ii) Element ~~stiffness~~, stress, σ
iii) support Reactions,

$E = 2\times10^5\ N/mm^2$ $P = 400KN$ $A_3 = 600mm^2$.

Find $u = ?$ $\sigma_i = ?$ $R = ?$.

Step 1:-



Step 2:-

Element stiffness Matrix for (1),

$$\frac{A_1E_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

$$\frac{300\times2\times10^5}{200}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}.$$

$$\Rightarrow 3\times10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}.$$

Ele (2).,

$$\frac{A_2E_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}F_2\\ F_3\end{Bmatrix}$$

$$3\times10^5\begin{bmatrix}1 & -\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}F_2\\ F_3\end{Bmatrix}.$$

$$\Rightarrow \quad 1\times10^5 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\Rightarrow \quad 1\times10^5 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}.$$

Ele (3) :-

$$\frac{A_3 E_3}{l_3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}.$$

$$\frac{600\times2\times10^5}{200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

$$6\times10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}.$$

$$\Rightarrow \quad 3\times10^5 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}.$$

Global Stiffness Matrix:-

$$3\times10^5 \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1+1 & -1 & 0 \\ 0 & -1 & 1+2 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

Here, $u_1 = u_4 = 0$

$F_2 = 400\times10^3$ N

$$3\times10^5 \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_2 = 400\times10^3 \\ F_3 = ? \end{bmatrix}$$

$$6\times10^5 u_2 - 3\times10^5 u_3 = 400\times10^3$$

$$-3\times10^5 u_2 + 9\times10^5 u_3 = 0$$

$\underline{u_1 = 0}$ $\quad$ $\underline{u_2 = 0.8}$

$\underline{u_4 = 0}$ $\quad$ $\underline{u_3 = 0.26}$

Stress (1) :-

$$\sigma_1 = \frac{E_1 \times (U_2 - U_1)}{l_1}$$

$$= \frac{2 \times 10^5 \times (0.88 - 0)}{200} = 800 \ N/mm^2$$

$$\sigma_2 = \frac{2 \times 10^5 \times (0.26 - 0.8)}{200}$$

$$= -533 \ N/mm^2.$$

$$\sigma_3 = \frac{E_1 \times (U_4 - U_3)}{l_3}$$

$$= \frac{2 \times 10^5 \times (-0.26)}{200}$$

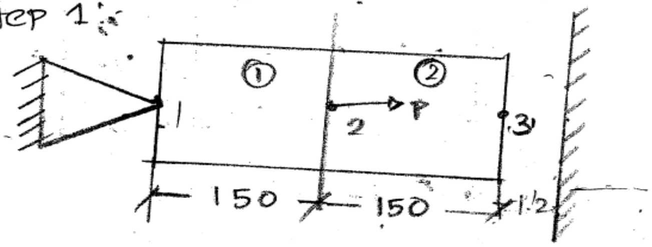
$$= -260 \ N/mm^2.$$

R_1 & R_2.

2013 6)

A Rod is subjected to Axial load P = 600KN is applied as shown in fig. Divide the Domain into two elements. Determine :- (i) Displacement at each load. (ii) stresses in each element, (iii) Reaction at Each Nodal points. Take $A = 250 \text{mm}^2$, $E = 2 \times 10^5 \text{ N/mm}^2$. $P = 600 \text{KN}$.

Step 1 :-



Step 2 :- For verification,

$$\delta = \frac{PL}{AE} = \frac{600 \times 10^3 \times 150}{250 \times 2 \times 10^5} = 1.8 \text{ mm}.$$

$$> 1.2 .$$

(Now wall is fixed with 3rd element)

$$u_3 = 0 .$$

Step 3 :-

Element Stiffness Matrix,

(1),

$$\frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} .$$

$$\frac{250 \times 2 \times 10^5}{150} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} .$$

$$3.3 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} .$$

Ele (2),

$$\frac{250 \times 2 \times 10^5}{150} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_2 \\ F_3 \end{bmatrix}$$

$$3.3 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_2 \\ F_3 \end{bmatrix} .$$

Global stiffness Matrix,

$$3.33\times10^{5}\begin{bmatrix}1 & -1 & 0\\ -1 & 1+1 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\\ F_3\end{bmatrix}$$

$u_1 = 0$, $u_3 = 0$, $F_2 = 600\times10^3$,

$F_1 = F_3 = 0$.

$$3.33\times10^{5}\begin{bmatrix}\cancel{1} & \cancel{-1} & \cancel{0}\\ \cancel{-1} & 2 & \cancel{-1}\\ \cancel{0} & \cancel{-1} & \cancel{1}\end{bmatrix}\begin{bmatrix}\cancel{0}\\ u_2\\ \cancel{0}\end{bmatrix}=\begin{bmatrix}\cancel{0}\\ 600\times10^3\\ \cancel{0}\end{bmatrix}$$

$$3.3\times10^{5}\times 2u_2 = 600\times10^{3}$$

$$u_2 = 0.9.$$

(i) $u_1 = 0$, $u_2 = 0.9$, $u_3 = 0$.

(ii) Stress at (1):-

$$\sigma_1 = \frac{E_1\times(u_2-u_1)}{l_1}$$

$$= \frac{2\times10^{5}\times(0.9)}{150}$$

$$= 1200\ \text{N/mm}^2.$$

Stress at (2):- $\sigma_2 = E_1\times(u_3-u_2)/l_2$

$$= 2\times10^{5}\times(-0.9)/150$$

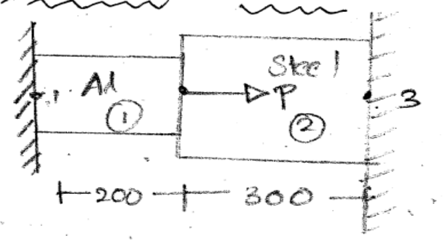
$$= -1200\ \text{N/mm}^2.$$

(iii) To find R:-

$$[K]\{u^*\}-\{F\}=\{R\}.$$

10/14

Temperature Effect



(7) An Axial load of 4×10^5 N is acting at Temp 30°c to the rod as shown. The Temp is raised to 60°c. Find.

i) Assemble K and F Matrix.

ii) Loaded displacement (u, u_2, u_3)

iii) stress in each Material (σ_1, σ_2).

iv) Reaction Force (R_1, R_2, R_3).

G:-

$A_1 = 1000\,mm^2$

$E_1 = 0.7\times10^5\,N/mm^2$.

$\alpha_1 = 23\times10^{-6}\,°c$

$A_2 = 1500\,mm^2$

$E_2 = 2\times10^5\,N/mm^2$

$\alpha_2 = 12\times10^{-6}\,/°c$.

$P = 4\times10^5\,N$

$t_1 = 30°c$

$t_2 = 60°c$

To find:-

i) $K = ?$

$F = ?$

ii) u_1, u_2, u_3

iii) σ_1, σ_2, iv) R_1, R_2, R_3.

S:-

The Finite element eqn for one dimensional two noded bar element is,

$$\frac{A_1E}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix}=\begin{Bmatrix}F_1\\ F_2\end{Bmatrix}.$$

For ele ①:-

The Element eqn,

$$\frac{A_1 E_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix}=\begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

$$\Rightarrow \frac{1000\times 0.7\times 10^5}{200}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\end{bmatrix}$$

$$\Rightarrow 35\times 10^{5}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\end{bmatrix} \Rightarrow 10^5\begin{bmatrix}3.5 & -3.5\\ -3.5 & 3.5\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

Ele ②:-

$$\frac{A_2 E_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}$$

$$\Rightarrow \frac{1500\times 2\times 10^5}{300}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}$$

$$\Rightarrow 10\times 10^{5}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}.$$

$$\text{Ele (1)} \Rightarrow 1\times 10^4\begin{bmatrix}3.5 & -3.5\\ -3.5 & 3.5\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\end{bmatrix}.$$

$$\text{Ele (2)} \rightarrow 1\times 10^5\begin{bmatrix}10 & -10\\ -10 & 10\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}$$

Global stiffness Matrix.

$$\Rightarrow 1\times 10^5\begin{bmatrix}3.5 & -3.5 & 0\\ -3.5 & 13.5 & -10\\ 0 & -10 & 10\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\\ F_3\end{bmatrix}$$

Assembling the F Matrix,

W.K.T, the load vector F

$$\{F\} = E \cdot A \cdot \alpha \cdot \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}.$$

Ele (1) $\Rightarrow$ $$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = E_1 A_1 \alpha_1 \Delta T \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

$$= 0.7\times10^5 \times 1000 \times 23\times10^{-6} \times(60-30) \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$=(1\times10^5)\times0.7\times1000\times23\times10^{-1}\times30 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= 4830 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= (1\times10^5) \begin{bmatrix} -0.483 \\ 0.4.83 \end{bmatrix}$$

Ele (2) $\rightarrow$ $$\begin{bmatrix} F_2 \\ F_3 \end{bmatrix} = 2\times10^5 \times 1500 \times 12\times10^{-6} \times 30 \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

$$= 108000 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= 1\times10^5 \begin{bmatrix} -1.08 \\ 1.08 \end{bmatrix}$$

Global Force vector

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = 1\times10^5 \begin{bmatrix} -0.483 \\ -0.597+4\times10^5 \\ 1.08 \end{bmatrix}$$

From the Figure we know that the axial load 4×10^5 N is acting on Node (2) so add 4×10^5

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = 1\times10^5 \begin{bmatrix} -0.483 \\ 3.403 \\ 1.08 \end{bmatrix}.$$

W.K.T,

[illegible]

$$1\times10^5 \begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} -0.483 \\ 399999.40 \\ 1.08 \end{bmatrix}$$

$$1\times10^5 \times 13.5\, u_2 = 399999.403$$

$u_1 = 0$, $u_2 = 0.2520$, $u_3 = 0$

(i) Stress at (1):-

$$\sigma_1 = \left[\frac{E_1 \times (u_2 - u_1)}{l_1}\right] - E_1 \alpha_1 \Delta T$$

$$= \left[\frac{0.7\times10^5 \times (0.2520)}{200}\right] - 0.7\times10^5 \times 23\times10^{-6} \times 30$$

$$= 89.9 \ \frac{N}{mm^2}$$

(ii) Stress at (2):- $\sigma_2 = \dfrac{E_2 \times (u_3 - u_2)}{l_1} - E_2 \alpha_2 \Delta T$

$$= \left[\frac{2\times10^5 \times (+0.2520)}{300}\right] - 2\times10^5 \times 12\times10^{-6} \times 30$$

$$= -240 \ \frac{N}{mm^2}$$

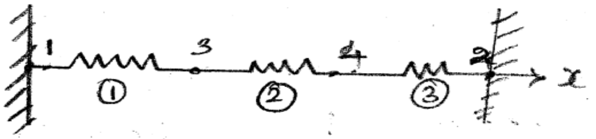
Resistance R_1, R_2, R_3 :- $[k]\,[u^*] - [F] = [R]$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{bmatrix} 0 \\ 0.2520 \\ 0 \end{bmatrix} - \begin{bmatrix} -0.483\times10^5 \\ 3.403\times10^5 \\ 1.08\times10^5 \end{bmatrix}$$ wrong.

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} 0 - 0.882 + 0 \\ 0 + 3.402 - 0 \\ 0 - 2.52 + 0 \end{bmatrix} - \begin{bmatrix} -0.483\times10^5 \\ 3.403\times10^5 \\ 1.08\times10^5 \end{bmatrix}$$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} 48299.11 \\ 0 \\ -108002.52 \end{bmatrix}$$

2) For the bar Assembly shown in Fig. Determine the i) Nodal displaceme ii) Global stiffness Matrix iii) Reaction Force. The Nodal Force F_4 is 500k $k_1 = 100 \text{ kN/m}$, $k_2 = 200 \text{ kN/m}$, $k_3 = 300 \text{ kN/m}$



S:- The Finite Element Eqn for spring system is given by,

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

For ele (1):- Finite Ele eqn,

$$\begin{bmatrix} F_1 \\ F_3 \end{bmatrix} = k_1 \begin{array}{cc|c} 1 & 3 & \\ 1 & -1 & 1 \\ -1 & 1 & 3 \end{array} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_3 \end{bmatrix} = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix}$$

For ele (2):-

$$\begin{bmatrix} F_3 \\ F_4 \end{bmatrix} = k_2 \begin{array}{cc|c} 3 & 4 & \\ 1 & -1 & 3 \\ -1 & 1 & 4 \end{array} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$

$$\begin{bmatrix} F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$

For ele (3):-

$$\begin{bmatrix} F_4 \\ F_2 \end{bmatrix} = \begin{bmatrix} 300 & -300 \\ -300 & 300 \end{bmatrix} \begin{bmatrix} u_4 \\ u_2 \end{bmatrix}.$$

Assemble Global stiffness Matrix,

$$\begin{array}{cccc|c} 1 & 2 & 3 & 4 & \\ \hline 100 & 0 & -100 & 0 & 1 \\ 0 & 300 & 0 & -300 & 2 \\ -100 & 0 & 300 & -200 & 3 \\ 0 & -300 & -200 & \begin{smallmatrix}200\\+\\300\end{smallmatrix} & 4 \end{array} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

Apply the boundary condition

Here, Node position changes where 1 & 2 are Fixed so, 1 & 2 column and row are cancelled

$$\begin{bmatrix} \cancel{100} & \cancel{0} & \cancel{-100} & \cancel{0} \\ \cancel{0} & \cancel{300} & \cancel{0} & \cancel{-300} \\ \cancel{-100} & \cancel{0} & 300 & -200 \\ \cancel{0} & \cancel{-300} & -200 & 500 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 500 \end{bmatrix}$$

$$\begin{bmatrix} 300 & -200 \\ -200 & 500 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 500 \end{bmatrix}$$

$$300u_3 - 200u_4 = 0. \quad F_3 = 0.$$

$$-200u_3 + 500u_4 = 500.$$

$$\boxed{u_3 = 0.9091m}$$

$$\boxed{u_4 = 1.364m}$$

To find R_1, R_2, R_3, R_4 :-

$$[K]\ [u^*] - [F] = [R].$$

$$\begin{bmatrix} 100 & 0 & -100 & 0 \\ 0 & 300 & 0 & -300 \\ -100 & 0 & 300 & -200 \\ 0 & -300 & -200 & 500 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.9091 \\ 1.364 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 500 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}$$

$R_1 = -90.9$ kN

$* \; 100 \times 0 + 0 \times 0 - 100 \times 0.9091 + 0 \times 1.364 - 0 = R_1$

$\boxed{\Rightarrow R_1 = -90.9 \text{ KN}}$

$R_2 = -409.2$ kN

$* \; 0 \times 0 + 300 \times 0 + 0 \times 0.901 - 300 \times 1.364 - 0 = R_2$

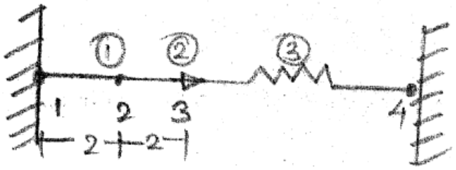
$\boxed{\Rightarrow R_2 = -409.2 \text{ KN}}$

$R_3 = 0$

$R_4 = 0$

— x —

3) $E = 70$ Gpas, $A = 2 \times 10^{-4} m^2$, $k = 2000$ KN/m, $P = 8$ KN. $\frac{1}{2}$



$\frac{1}{2}$

G:-

$E = 70$ Gpas $= 70 \times 10^9$ pas

$A = 2 \times 10^{-4} m^2$

$K = 2000$ KN/m

$P = 8$ KN.

To find:-

SHAPE FUNCTIONS .

8/10/14.

2.14. Consider a taper steel plate of uniform thickness t = 25mm . . The youngs Modulus of Plate $E = 2\times10^5$ N/mm^2 and weight density , $\rho = 0.82 \times 10^{-4}$ N/mm^3. In Addition to its self weight , the plate is subjected to a point load P = 100N at its Mid point . Calculate the following by Modelling the plate with two finite elements .

2.22 (+) 2.23

(i) Global Force vector {F}.

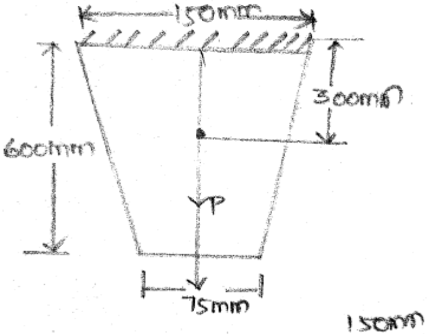
(ii) Global stiffness Matrix [k] .

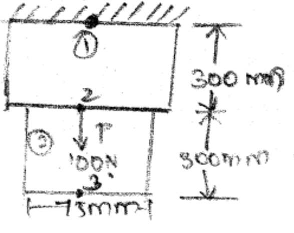
(iii) Displacement in each element .

(iv) stress in each element .

(v) Reaction force at support .

[redacted]





Area at Node 1, A_1 = Width × thickness

$= W_1 \times t_1$

$= 150 \times 25 = 3750 mm^2$.

Area at Node 2, $A_2 = W_2 \times t_2$

$= \left(\frac{W_1 + W_3}{2}\right) \times t_2$

$= \left(\frac{150 + 75}{2}\right) \times 25$

$[t_1 = t_2 = t_3 = 25mm]$

$\underbrace{A_2}= \underbrace{2812.5 mm^2}$.

Area at Node 3, $A_3 = W_3 \times t_3$

$= 75 \times 25$

$= 1875 mm^2$.

Average area of element (1):

$\bar{A}_1$ = Area at node 1 + Area at Node 2 / 2

$\bar{A}_1 = 3281.25 mm^2$.

At element (2):-

$\bar{A}_2$ = Area at Node, 2 + Area at Node, 3 / 2

$= 2812.5 + 1875/2$

$\bar{A}_2 = 2343.75 mm^2$.

Youngs Modulus $E = 2 \times 10^5 N/mm^2$.

Weight density $P = 0.82 \times 10^{-4} N/mm^2$

Length $l = 300mm$.

To find:-

i) Global force vector (F):

ii) Global stiffness Matrix (K).

iii) Displacement in each element.

iv) Stresses in each element.

v) Reaction force at the support.

Sol:- The steel plate is subjected to self weight so, we Have to find the body Force acting at Nodal points 1,2 and 3.

W.K.T,

Body Force vector $\{F\} = \frac{\rho A l}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$

For element (1):-

$$\text{Force vector } \begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{\rho_1 A_1 l}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \frac{0.82\times10^{-4}\times 3281.25\times300}{2}\begin{bmatrix}1\\1\end{bmatrix}.$$

$$= 40.359\times\begin{bmatrix}1\\1\end{bmatrix}.$$

$$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \begin{Bmatrix}40.359\\40.359\end{Bmatrix}.$$

Element (2):-

$$\begin{bmatrix}F_2\\F_3\end{bmatrix} = \frac{\rho_2 A_2 l_2}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= 0.82\times10^{-4}\times 2343.75\times300/_2 \times\begin{bmatrix}1\\1\end{bmatrix}$$

$$= 28.828\times\begin{bmatrix}1\\1\end{bmatrix}$$

$$\begin{bmatrix}F_2\\F_3\end{bmatrix} = \begin{bmatrix}28.828\\28.828\end{bmatrix}.$$

Assembling a Force Vector,

$$\begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix} = \begin{bmatrix}40.359\\40.359+28.828\\28.828\end{bmatrix} = \begin{bmatrix}40.359\\69.187\\28.828\end{bmatrix}$$

A Point load 100 kN acts at Node 2,

So, Add 100N of F_2 vector,

$$\begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix} = \begin{bmatrix}40.359\\69.187+100\\28.828\end{bmatrix}$$

Global Force vector $= \begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix} = \begin{bmatrix}40.359\\169.187\\28.828\end{bmatrix}$.

Finite element eqn for one dimension,

$$\begin{bmatrix}F_1\\F_2\end{bmatrix} = \frac{AE}{l}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}.$$

Element (1), Nodes (1, 2),

$$\frac{A_1 E}{l}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix} = \begin{bmatrix}F_1\\F_2\end{bmatrix}$$

$$\frac{3281.25\times2\times10^5}{300}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix} = \begin{bmatrix}F_1\\F_2\end{bmatrix}$$

$$10.937\times2\times10^5\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix} = \begin{bmatrix}F_1\\F_2\end{bmatrix}$$

$$2\times10^5\begin{bmatrix}10.937&-10.937\\-10.937&10.937\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix} = \begin{bmatrix}F_1\\F_2\end{bmatrix} \quad -(1)$$

Element (2), Nodes (2,3)

$$2343.75\times2\times10^5/300\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}u_2\\u_3\end{bmatrix} = \begin{bmatrix}F_2\\F_3\end{bmatrix}$$

$$7.8125\times2\times10^5\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}u_2\\u_3\end{bmatrix} = \begin{bmatrix}F_2\\F_3\end{bmatrix} \quad -(2)$$

Assemble F.e.eqn (1) and (2)

$$\Rightarrow 2\times10^5\begin{bmatrix}10.937&-10.937&0\\-10.937&10.937+7.8125&-7.8125\\0&-7.8125&7.8125\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix}$$

$$2\times10^5\underbrace{\begin{bmatrix}10.937&-10.937&0\\-10.937&18.749&-7.8125\\0&-7.8125&7.8125\end{bmatrix}}_{K}\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix} \quad \to (3)$$

Apply the boundary conditions, (i,e) at Node 1, displacement $u_1=0$, sub u_1, F_1, F_2 and F_3 in (3).

$$2\times10^5 \begin{bmatrix}10.937 & -10.937 & 0\\ -10.937 & 18.749 & -7.8125\\ 0 & -7.8125 & 7.8125\end{bmatrix}\begin{bmatrix}0\\ u_2\\ u_3\end{bmatrix} = \begin{bmatrix}40.359\\ 169.187\\ 28.828\end{bmatrix}$$

Neglect 1st row and First Column

$$2\times10^5 \begin{bmatrix}18.749 & -7.8125\\ -7.8125 & 7.8125\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix} = \begin{bmatrix}169.187\\ 28.828\end{bmatrix}$$

$$2\times10^5\,(18.749u_2 - 7.8125u_3) = 169.187 \quad -(4)$$

$$2\times10^5\,(-7.8125u_2 + 7.8125u_3) = 28.828 \quad -(5)$$

Solving $2\times10^5\,(0.936)u_2 = 198.05$

$$u_2 = 9.053\times10^{-5}\,\text{mm}.$$

Sub in. (4)

$$2\times10^5\,[18.749\,(9.053\times10^{-5}) - 7.8125u_3) = 169.187$$

$$18.749\times9.053\times10^{-5} - 7.8125u_3 = 8.459\times10^{-4}$$

$$-7.8125u_3 = -8.514\times10^{-4}$$

$$u_3 = 10.898\times10^{-5}\,\text{mm}.$$

W.K.T, Stress $\sigma = E\frac{du}{dx}$.

Element (1):- $\sigma_1 = E\times\frac{u_2-u_1}{l_1}$

$$= 2\times10^5\times\frac{(9.053\times10^{-5})}{300}$$

$$\sigma_1 = 0.060\ \text{N/mm}^2.$$

Element (2):- $\sigma_2 = E\times(u_3-u_2)/l_2$

$$= 2\times10^5\times(10.898\times10^{-5} - 9.053\times10^{-5})/300$$

$$\sigma_2 = 0.0123\ \text{N/mm}^2$$

Reaction Force :-

$$\{R\} = \{K\}\ [u] - [F]$$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = 2\times10^5 \begin{bmatrix} 10.937 & -10.937 & 0 \\ -10.937 & 18.749 & -7.8125 \\ 0 & -7.8125 & 7.8125 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} - \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$= 2\times10^5 \begin{bmatrix} 10.937 & -10.937 & 0 \\ -10.937 & 18.749 & -7.8125 \\ 0 & -7.8125 & 7.8125 \end{bmatrix} \begin{bmatrix} 0 \\ 9.053\times10^{-3} \\ 10.898\times10^{-5} \end{bmatrix} - \begin{bmatrix} 40.359 \\ 169.187 \\ 28.828 \end{bmatrix}$$

$$= \begin{bmatrix} -198.02 \\ 169.187 \\ 28.828 \end{bmatrix} - \begin{bmatrix} 40.359 \\ 169.187 \\ 28.828 \end{bmatrix}$$

$$= \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} -238.379 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 = -238.379$ N.

$R_2 = 0$ N

$R_3 = 0$ N

Verification :- $R_1 + R_2 + R_3 = -238\ 379$ N.

Applied Forces :- $= F_1 + F_2 + F_3$

$= 40.359 + 169.187 + 28.828$

$= 238.37$ N.

15/10/14. (7)(8M) Unit - IV .

NUMERICAL INTEGRATION

No. of points	Location x_i	Corresponding Weights (w_i)
1 .	$x_1 = 0.000$	2.0000 .
2 .	$x_1, x_2 = \pm 0.5773$ [illegible]	[illegible] 1.0000
3 .	$x_1, x_3 = \pm 0.7745$ $x_2 = 0.0000$	$5/9 = 0.5555$ $8/9 = 0.8888$
4 .	$x_1, x_4 = \pm 0.86113$ $x_2, x_3 = \pm 0.3399$	0.3478 0.6521

① Evaluate $\int_{-1}^{1} (x^4 + x^2)\, dx$. By Applying 3 Point Gaussian quadrature .

G:-

$$\int_{-1}^{1} (x^4 + x^2)\, dx$$

$$\Rightarrow f(x) = (x^4 + x^2)$$

S:-

W.K.T, For 3 point Gaussian quadrature ,

$x_1 = 0.7745$ $\quad W_1 = 0.5555$

$x_2 = 0.0000$ $\quad W_2 = 0.8888$

$x_3 = -0.7745$ $\quad W_3 = 0.5555$

W.K.T, $f(x) = (x^4 + x^2)$

$f(x_1) = ((0.7745)^4 + (0.7745)^2) = 0.96$

$f(x_2) = 0.$

$f(x_3) = ((-0.7745)^4 + (-0.7745)^2) = 0.96$

$\therefore \quad f(x_1).W_1 = (0.96)\ (0.5555) = 0.53 \rightarrow (1)$

$f(x_2).W_2 = (0) = 0$

$f(x_3).W_3 = (0.96)\ (0.5555) = 0.53 \rightarrow (3)$

Adding (1) (2) and (3),

① + ② + ③

= 1.066. //.

Verification, Integrating $\int_{-1}^{1} (x^4 + x^2)\, dx$

$$\Rightarrow \left[\frac{x^5}{5} + \frac{x^3}{3}\right]$$

$$\Rightarrow \left[\left(\frac{1}{5} + \frac{1}{3}\right) - \left(\frac{-1}{5} - \frac{1}{3}\right)\right]$$

$$\Rightarrow \frac{1}{5} + \frac{1}{3} + \frac{1}{5} + \frac{1}{3} \Rightarrow \frac{2}{5} + \frac{2}{3} \Rightarrow 1.06 \ //$$

— x —

② Evaluate $\int_{-1}^{1} (x^4 - 3x + 7).dx$.

S:- W.K.T the given Integral is a polynomial of order 4, so For Exact integration $2n - 1 = 4 \Rightarrow n = 2.5.$
TAKE $n = 3$.

$$\int_{-1}^{1} (x^4 - 3x + 7)\, dx$$

$$f(x) = (x^4 - 3x + 7)\, dx.$$

W.K.T, for 3 point gaussian quadrature,

$x_1 = 0.7745$ $\quad W_1 = 0.5555$

$x_2 = 0$ $\quad W_2 = 0.8888$

$x_3 = -0.7745$ $\quad W_3 = 0.5555$

W.K.T, $f(x) = x^4 - 3x + 7$

$f(x_1) = (0.7745)^4 - 3(0.7745) + 7 = 5.04$

$f(x_2) = 7$

$f(x_3) = (-0.7745)^4 - 3(-0.7745) + 7$

$= 9.68$

$f(x_1)(W_1) = (5.04)(0.5555) = 2.79 \rightarrow (1)$

$f(x_2)(W_2) = (7)(0.8888) = 6.22 \rightarrow (2)$

$f(x_3)(W_3) = (9.68)(0.5555) = 5.37 \rightarrow (3)$

Adding ① + ② + ③;

$= 2.79 + 6.22 + 5.37$

$= 14.4$

Verification,

$$\int_{-1}^{1} (x^4 - 3x + 7)\,dx$$

$$= \cancel{\left[\frac{x^5}{5} - 3\right]_{-1}^{1}}$$

$$= \left[\frac{1}{5} - 3\right] - \cancel{\left[\frac{-1}{5} - 3\right]}$$

$$= \frac{1}{5} - \cancel{3 + \frac{1}{5} + 3}$$

$=$

$$= \left[\frac{x^5}{5} - \frac{3x^2}{2} + 7x\right]_{-1}^{1}$$

$$= \left[\frac{1}{5} - \frac{3}{2} + 7\right] - \left[\frac{-1}{5} - \frac{3}{2} - 7\right]$$

$$= \left[\frac{1}{5} - \cancel{\frac{3}{2}} + 7 + \frac{1}{5} + \cancel{\frac{3}{2}} + 7\right] = 14.4$$

Hence Result

— x —

③ Evaluate $\int_{-1}^{1} \cos\frac{x}{2}\,dx$ by applying 3 point gaussian quadrature.

G:-

$$\int_{-1}^{1} \cos\frac{x}{2}\,dx$$

$$f(x) = \cos x/2$$

S:-

W.K.T, For 3 point gaussian quadrature,

$x_1 = 0.7745$, $W_1 = 0.5555$

$x_2 = 0$, $W_2 = 0.8888$

$x_3 = -0.7745$, $W_3 = 0.5555$

W.K.T, $f(x) = \cos\frac{x}{2}$

$f(x_1) = \cos\frac{(0.7745)}{2} = 0.9259$

$f(x_2) = \cos\frac{(0)}{2} = 1$

$f(x_3) = \cos\frac{(-0.7745)}{2} = 0.9259$

$f(x_1)(W_1) = 0.5144 \rightarrow (1)$ $f(x_3) = +0.5144 \rightarrow (3)$

$f(x_2)(W_2) = 0.8888 \rightarrow (2)$

Adding (1) + (2) + (3) = 1.9976.

Verification,

$$\int_{-1}^{1} \cos\frac{x}{2}\, dx.$$

$$= \left[-\frac{\sin x}{2}\right]_{-1}^{1} = \left[\frac{\sin 1}{2} + \sin\left(\frac{-1}{2}\right)\right]$$

$$= 1.9176$$

— × —

4) Integrate the function,

$f(r) = \int_{-1}^{1} (1+r+r^2+r^3)\cdot dr$. solve by using gaussian Quadrature.

$$2n-1 = 3$$

$$2n = 3+1$$

$$n = \frac{4}{2} = 2.$$

G:-

$$\int_{-1}^{1} (1+r+r^2+r^3)\cdot dr.$$

$$\Rightarrow \quad f(x) = (1+r+r^2+r^3).$$

W.K.T, for 2 point gaussian Quadrature,

$x_1 = 0.5773$, $\quad W_1 = 1.0000$

$x_2 = -0.5773$, $\quad W_2 = 1.0000$

W.K.T, $f(r) = (1+r+r^2+r^3)$.

$f(x_1) = (1+0.5773+(0.5773)^2+(0.5773)^3)$

$= 2.10$.

$f(x_2) = (1(-0.5573)-(0.5773)^2-(0.5773)^3)$

$= 0.56$

$f(x_1) \cdot w_1 = (2.10)(1.000) = 2.10 \rightarrow (1)$

$f(x_2) \cdot w_2 = (0.56)(1.000) = 0.56 \rightarrow (2)$

Adding (1)+(2),

$= 2.66$.

verification,

$$\int_{-1}^{1} (1+r+r^2+r^3)\,dr$$

$$\left(r+\frac{r^2}{2}+\frac{r^3}{3}+\frac{r^4}{4}\right)_{-1}^{1}$$

$$= \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(-1+\frac{-1^2}{2}+\frac{-1^3}{3}+\frac{-1^4}{4}\right)$$

$$= 2.0833-(-0.5833)$$

$$= 2.6666 \;//$$

verified //.

— x —

5) $\int_{-1}^{1} \frac{\cos x}{1-x^2}\, dx$. By Applying 3 point gaussian quadrature.

G:- $\int_{-1}^{1} \frac{\cos x}{1-x^2}$.

$\Rightarrow f(x) = \frac{\cos x}{1-x^2}$.

W.K.T, for 3 point gaussian quadrature,

$x_1 = +0.7745$, $W_1 = 0.5555$

$x_2 = 0$ $W_2 = 0.8888$

$x_3 = -0.7745$, $W_3 = 0.5555$.

$f(x) = \frac{\cos x}{1-x^2}$.

$f(x_1) = \frac{\cos(0.7745)}{1-(0.7745)^2} = 1.48$.

$f(x_2) = \cos(0) = 1$

$f(x_3) = \frac{\cos(-0.7745)}{1-(-0.7745)^2} = 1.48$.

$f(x_1) \cdot W_1 = (1.48)(0.5555) = 0.98$.

$f(x_2) \cdot W_2 = (1)(0.8888) = 0.8888$.

$f(x_3) \cdot W_3 = (1.48)(0.5550) = 0.98$ ..

$\Rightarrow 1.38 + 0.8888 + 1.38 \Rightarrow 1.96$.

Verification :-

$$\int_{-1}^{1} \frac{\cos x}{1-x^2} \qquad \frac{u}{v} = \frac{v(u') - u(v')}{v^2}$$

$$= \frac{\left[(1-x^2)\left(\frac{\sin x^2}{2}\right) - \cos\left(-\frac{x^3}{3}\right)\right]_{-1}^{1}}{(1-x^2)^2}$$

$$= \frac{\left[(1-x^2)\left(\frac{\sin x^2}{2}\right) - \cos\left(-\frac{x^3}{3}\right)\right]_{-1}^{1}}{(1-x^2)^2}$$

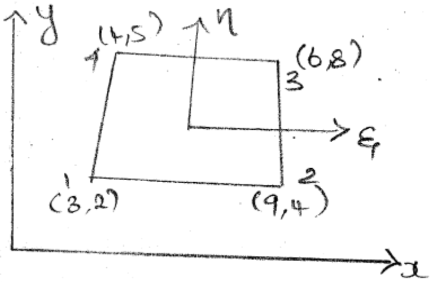
$$= \frac{\left[(1-1)\left(\frac{\sin}{2}\right) - \cos\left(-\frac{1}{3}\right)\right]}{(1-1)^2}$$

$$= \left[\frac{(0)\left(\frac{\sin}{2}\right) - \cos\left(-\frac{1}{3}\right)}{(0)}\right]$$

$=$

16/10/14. Iso Parametric :-

① Evaluate the cartesian co-ordinate of the point (P) which as local co-ordinate $\xi = 0.6$, $\eta = 0.8$ as shown in fig :-



G :- The Natural co-ordinate at point P is, The cartesian co-ordinates are of point 1,2,3 & 4 are given by.

$(x_1, y_1) = (3,2)$

$(x_2 y_2) = (9,4)$

$(x_3 y_3) = (6,8)$

$(x_4 y_4) = (4,5)$

S :-

W.K.T, shape fun for. coordilateral is,

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

Now, we can sub the ξ & η values in N_1, N_2, N_3 & N_4.

$N_1 = \frac{1}{4}\ (1-0.6)\ (1-0.8) = 0.02.$

$N_2 = \frac{1}{4}\ (1+0.6)\ (1-0.8) = 0.08.$

$N_3 = \frac{1}{4}\ (1+0.6)\ (1+0.8) = 0.72\ .$

$N_4 = \frac{1}{4}\ (1-0.6)\ (1+0.8) = 0.18\ .$

Now find out x, y :-

$x = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4\ .$

$= 0.02x_3 + 0.08\times 9 + 0.72\times 6 + 0.18\times 4$

$x = 5.82$

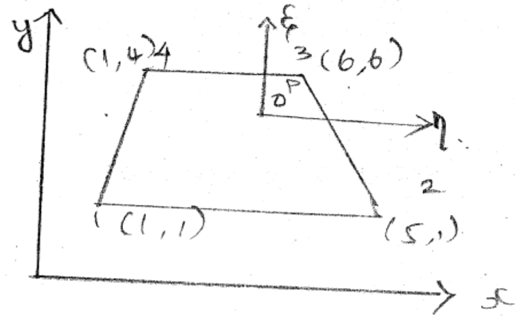
$y = N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4$

$= 0.02\times 2 + 0.08\times 4 + 0.72\times 8 + 0.18\times 5$

$y = 7.02\ .$

—x—

② For the isoparametric four node co-ordilateral element shown in fig. Determine co-ordinate at point (P) which is local co-ordinates, $\xi = 0.5,\ \eta = 0.5$. (



G:-

$(x_1\ y_1) = (1,1)$ $\xi = 0.5$.

$(x_2\ y_2) = (5,1)$ $\eta = 0.5$

$(x_3\ y_3) = (1,4).$

S:- Now find out N_1, N_2, N_3, N_4.

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta) = 1/4\,(1-0.5)(1-0.5) = 0.06$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta) = 1/4\,(1+0.5)(1-0.5) = 0.18$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta) = 1/4\,(1+0.5)(1+0.5) = 0.56$$

$$N_4 = 1/4\,(1-\xi)(1+\eta) = 1/4\,(1-0.5)(1+0.5) = 0.18.$$

Now find out x, y.

$$x = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4$$

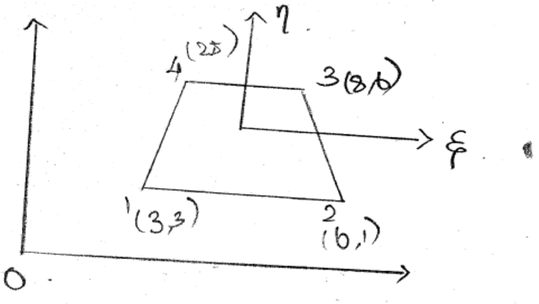
$$= 0.06\times1 + 0.18\times5 + 0.56\times6 + 0.18\times1 = 4.5.$$

$$y = N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4$$

$$= 0.06\times1 + 0.18\times1 + 0.56\times6 + 0.18\times4 = 4.32.$$

—x—

③. For the isoparametric Quadrilateral Element shown in the fig. Determine the local coordinates of the point (P) which is cartesian coordinates (7, 4).



G:- $(x_1, y_1) = (3,1)$ $x = 7$

$(x_2, y_2) = (6,1)$ $y = 4$

$(x_3\ y_3) = (8,6)$

To find:- $\xi = ?$ $\eta = ?$

$(x_4\ y_4) = (2,5).$

Sol:- $x = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4.$

$y = N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4.$

sub values are x & y and $N_1, N_2, N_3, N_4, x_1, x_2, x_3, x_4$.

$$7 = \frac{1}{4}(1-\xi)(1-\eta)3 + \frac{1}{4}(1+\xi)(1-\eta)6 + \frac{1}{4}(1+\xi)(1-\eta)8 + \frac{1}{4}(1-\xi)(1+\eta)2 \longrightarrow ①$$

$$4 = \frac{1}{4}(1-\xi)(1-\eta)1 + \frac{1}{4}(1+\xi)(1-\eta)1 + \frac{1}{4}(1+\xi)(1+\eta)6 + \frac{1}{4}(1-\xi)(1+\eta)5 \longrightarrow ②.$$

$$\frac{1}{4}(1-\xi)(1-\eta)3 + \frac{1}{4}(1+\xi)(1-\eta)6 + \frac{1}{4}(1+\xi)(1-\eta)8 + \frac{1}{4}(1-\xi)(1+\eta)2 = 7.$$

$$② \times 3 \quad \underset{(-)}{\frac{1}{4}}(1-\xi)(1-\eta)3 + \underset{(-)}{\frac{1}{4}}(1+\xi)(1-\eta)3 + \underset{(-)}{\frac{1}{4}}(1+\xi)(1-\eta)18 + \underset{(-)}{\frac{1}{4}}(1-\xi)(1+\eta)5 \underset{(-)}{=12}$$

$$\frac{1}{4}\left[(1+\xi)(1-\eta)3 - (1+\xi)(1-\eta)10 - (1-\xi)(1+\eta)3\right] = -5.$$

① $\Rightarrow$

$$\frac{1}{4}\left[(1-\xi)(1-\eta)3 + (1+\xi)(1-\eta)6 + (1+\xi)(1+\eta)8 + (1-\xi)(1+\eta)2\right] = 7$$

$$3 - 3\eta - 3\xi + 3\xi\eta + 6 - 6\eta + 6\xi - 6\xi\eta + 8 + 8\eta + 8\xi + \eta\xi 8 + 2 + 2\eta - 2\xi$$

$$4 - 2\xi\eta = 28 - 19$$

$$\eta + 9\xi + 3\xi\eta = 9 \rightarrow ③.$$

similarly ② $\Rightarrow$

$$4 = \frac{1}{4}(1-\xi)(1-\eta) + \frac{1}{4}(1+\xi)(1-\eta) + 6\times\frac{1}{4}(1+\xi)(1+\eta) + 5\times\frac{1}{4}(1-\xi)(1+\eta)$$

$$4 = \frac{1}{4}(1-\eta-\xi+\eta\xi) + \frac{1}{4}(1-\eta+\xi-\eta\xi) + \frac{3}{2}(1+\eta+\xi+\eta\xi) + \frac{5}{4}(1+\eta-\xi-\xi\eta)$$

$$\Rightarrow 4 = \frac{1}{4} - \frac{\eta}{4} - \frac{\xi}{4} + \frac{\eta\xi}{4} + \frac{1}{4} - \frac{\eta}{4} + \frac{\xi}{4} - \frac{\eta\xi}{4} + \frac{3}{2} + \frac{3\eta}{2} + \frac{3\xi}{2} + \frac{3}{2}\eta\xi + \frac{5}{4} + \frac{5}{4}\eta - \frac{5}{4}\xi - \frac{5}{4}\xi\eta$$

$$\Rightarrow \frac{3}{4} = \frac{9}{4}\eta + \frac{1}{4}\xi + \frac{1}{4}\xi\eta$$

$$\Rightarrow 3 = 9\eta + \xi + \xi\eta \rightarrow ④$$

$$9 = \eta + 9\xi + 3\xi\eta \longrightarrow ③$$

$$3 = 9\eta + \xi + \xi\eta \longrightarrow ④$$

④ × 3

$$9 = 27\eta + 3\xi + 3\xi\eta$$

$$\underset{(-)}{9} = \underset{(-)}{\eta} + \underset{(-)}{9\xi} + \underset{(-)}{3\xi\eta}$$

$$0 = 26\eta - 6\xi$$

$$6\xi = 26\eta$$

$$\xi = 4.333\eta \text{. in } ③$$

$$9 = \eta + 9\times 4.333\eta + 3\times 4.333\eta^2$$

$$9 = \eta + 39\eta + 13\eta^2$$

$$9 = 40\eta + 13\eta^2$$

$$13\eta^2 + 40\eta - 9 = 0$$

$$\eta = -40 \pm \sqrt{(40)^2 - 4\times 13\times -9} \,/\, 2\times 13$$

$$\underline{\eta = 0.210587}$$

$G_q = 4.333(0.21084) = 0.9135$

— × —

25/10/14.

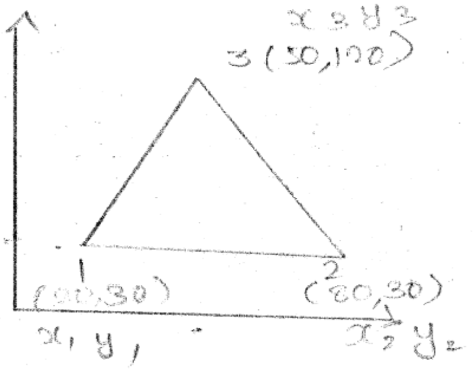
4. Determine the stiffness Matrix for the constant strain Triangular element as shown in Fig. Coordinates are given in mm. Assume Plane stress condition. Take E = 210 Gpa., t = 10mm., V = 0.25.

G:-

E = 210×10^9 pa.

t = 10mm.

V = 0.25.



To find:- $[K] = ?$

S:- W.K.T, stiffness Matrix

$$[K] = [B]^T \cdot [D] \cdot [B] \cdot A \cdot t \,.$$

Step 1:-

$$A = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} .$$

$$= \frac{1}{2} \; 1(x_2 y_3 - y_2 x_3) - x_1(y_3 - y_2) + y_1(x_3 - x_2) \,.\,.$$

$$= \frac{1}{2} \; ((80)(120) - (30)(50) - 20(120-30) + 30(50-80))$$

$$= 2700 \text{mm} .$$

Step-II:- Strain - Displacement Matrix [B]

$$\Rightarrow \frac{1}{2A} \begin{vmatrix} a_1 & 0 & a_2 & 0 & a_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & a_1 & r_2 & a_2 & r_3 & a_3 \end{vmatrix}.$$

$a_1 = y_2 - y_3 = 30 - 120 = -90$

$a_2 = y_3 - y_1 = 120 - 30 = 90$

$a_3 = y_1 - y_2 = 30 - 30 = 0$

$r_1 = x_3 - x_2 = 50 - 80 = -30$

$r_2 = x_1 - x_3 = 80 - 50 = 30$

$r_3 = x_2 - x_1 = 60$

$$[B] = \frac{1}{2(2700)} \begin{vmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & 60 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{vmatrix}.$$

$$= 5.55 \times 10^{-3} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

Step-III :- Stress- strain Relationship.

$$[D] = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1 - \frac{\nu}{2} \end{bmatrix}$$

E = 210 Gpa

$\downarrow$

$(2.1 \times 10^5 \, N/mm^2)$

$$= \frac{2.10 \times 10^5}{1 - (0.25)^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1 - 0.25}{2} \end{bmatrix}$$

(Taking 0.25)

$$[D] = .56 \times 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

step - IV :- $[D]\ [B]$

$$= \begin{matrix} 5.6[illegible]\times10^{+3} \\ \times 5.\underset{5}{5}\times10^{-3} \end{matrix} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$= 310.8 \begin{bmatrix} (-12+0+0) & (0-1+0) & (12+0+0) & (0-1+0) & (0+0+0) & (0+2+0) \\ (-3+0+0) & (0-4+0) & (3+0+0) & (0-4+0) & (0+0+0) & (0+8+0) \\ (0+0-1.5) & (0+0-4.5) & (0+0-1.5) & (0+0+4.5) & (0+0+3) & (0+0+0) \end{bmatrix}$$

$$= 310.8 \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}_{3\times6}$$

Step - V :- Find $[B]^T$.

$$= 5.55\times10^{-3} \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Step VI :-

$[B]^T\ [D]\ [B]$

$$= \begin{matrix} 5.55\times10^{-3} \\ \cancel{\times 5.6\times10^{3}} \\ \times 310.8 \end{matrix} \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

$$= 5.55\times10^{-3} \times 310.8 \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

$$= 1.72494 \begin{bmatrix} 36+0+1.5 & 3+0+4.5 & -36+0+1.5 & 3+0-4.5 & \begin{matrix} 0+0-3 \\ -6+0+0 \end{matrix} \\ 0+3+4.5 & 0+4+13.5 & 0-3+4.5 & 0+4-13.5 & \begin{matrix} 0+8+9 \\ 0-8+0 \end{matrix} \\ -36+0+1.5 & -3+0+4.5 & 36+0+1.5 & -3+0-4.5 & \begin{matrix} 0+0-3 \\ 6+0+0 \end{matrix} \\ 0+3-4.5 & 0+4-13.5 & 0-3-4.5 & 0+4+13.5 & \begin{matrix} 0+0+9 \\ 0-8+0 \end{matrix} \\ 0+0-2.5 & 0+0-9.0 & 0+0-3.0 & 0+0+9.0 & \begin{matrix} 0+0+6 \\ 0+0+0 \end{matrix} \\ 0-6+0 & 0-8+0 & 0+6+0 & 0-8+0 & \begin{matrix} 0+0+6 \\ 0+16+0 \end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix} \times 1.7249$$

Step VII :- $[B]^T [D] \cdot [B] \cdot A \cdot t$.

~~89.50~~

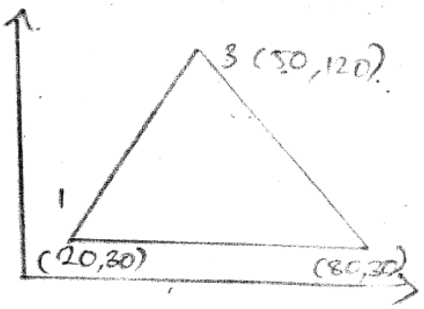
$$= 1.7249 \times 2700 \times 10 \cdot \begin{bmatrix} +37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

$$= 46572.3 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix} //$$

— × —

29/10/14.

2) For a Plane stress Element shown in Fig. the Nodal displacements are, $u_1 = 2mm$, $u_2 = 0.5mm$, $u_3 = 3mm$, $V_1 = 1mm$, $V_2 = 0mm$, $V_3 = 1mm$. Determine the element stress σ_x, σ_y, τ_{xy}, σ_1, σ_2, The Principle angle θ_p, Take $E = 210$ Gpa and $\nu = 0.25$ and thickness = 10mm. All the co-ordinates are in mm.



G:-

$E = 210$ Gpa.

$t = 10mm$.

$\nu = 0.25$.

S:-

W.K.T, Stiffness Matrix,

$$[K] = [B]^T \cdot [D]\,[B].$$

Step 1:-

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$= \frac{1}{2}(x_2y_3 - x_3y_2) - x_1(y_3 - y_2) + y_1(x_3 - x_2)$$

$$= 2700mm.$$

Step 2:-

Strain displacement Matrix [B]

$$\Rightarrow \frac{1}{2A} \begin{bmatrix} \alpha_1 & 0 & \alpha_2 & 0 & \alpha_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \alpha_1 & \gamma_2 & \alpha_2 & \gamma_3 & \alpha_3 \end{bmatrix}$$

$\alpha_1 = -90$, $\alpha_2 = 90$, $\alpha_3 = 0$, $\gamma_1 = -30$

$\gamma_2 = -30$, $\gamma_3 = 60$

$$[B] = 5.55 \times 10^{-3} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

Step - III. Stress - strain Relationship,

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$[D] = 56 \times 10^{3} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}.$$

Step - IV. $[D]\ [B]$.

$$= 5.6 \times 10^{3} \times 5.55 \times 10^{-3} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}.$$

$$= 310.8 \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & +3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

Step $\underline{V}$:- W.K.T , stress = [D] [B] {u}

$$= [D] \quad [B] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$= 310.8 \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = 310.8 \begin{bmatrix} -24 - 1 + 6 + 0 + 0 + 2 \\ -6 - 4 + 1.5 + 0 + 0 + 8 \\ -3.0 - 4.5 - 0.75 + 0 + 9 + 0 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = 310.8 \begin{bmatrix} -17 \\ -0.5 \\ -0.75 \end{bmatrix} = \begin{bmatrix} -5283.6 \\ -155.4 \\ 233.1 \end{bmatrix}$$

Step $\underline{VI}$:-

Max. Normal Stress (σ_{max})

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-5283.6 - 155.4}{2} + \sqrt{\left(\frac{-5283.6 + 155.4}{2}\right)^2 + (-233.1)^2}$$

$= -2719.5 +$

$\sigma_1 =$ -144.8263×10^3 N/mm^2

$\boxed{\sigma_1 = -0.144\ N/mm^2}$

Min Normal stress (σ_{min})

$$\Rightarrow \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \ldots$$

$$\sigma_2 = -2719.5 - \sqrt{\left(\frac{-5283.6 - 155.4}{2}\right)^2 + (-233.1)^2}$$

$$\boxed{\sigma_2 = -5.294 \times 10^3\ N/mm^2}$$

Principle angle $= \tan 2\theta_p = \dfrac{2 \cdot \tau_{xy}}{\sigma_x \cdot \sigma_y}$.

$$\therefore \theta_p = \tan^{-1}\left[\frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}\right].$$

$$\therefore \theta_p = \tan^{-1}\left[\frac{\tau_{xy}}{\sigma_x - \sigma_y}\right]$$

$$= \tan^{-1}\left[\frac{-233.1}{-5\ldots}\right]$$

$\boxed{\theta_p = 3^\circ}$. $-5283.6 + 155.4 \doteq 2^\circ 36''$.

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UNIT – IV

	D	T	P
* Shape Fn		✓	✓
* CST – 2D			✓
1) 2D			
2) stress			
3) Temp			
* Isoparametric		✓	✓
* NI			✓

UNIT – V (S. SENTHIL)

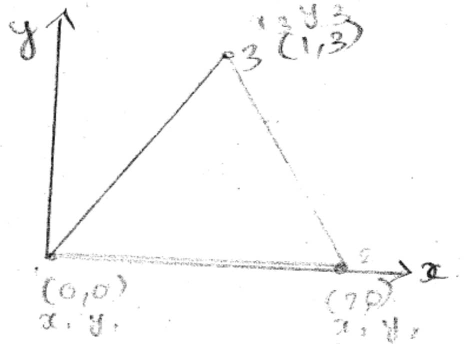
Pg. NO	PbM. NO
4.44	4.4
4.48	4.5
4.65	4.8
Heat Transfer ✓ 5.17	5.2, 5.1
✓ 5.23	5.3
✓ 5.31	5.5

UNIT – I

1.54	1.12

3. Calculate Element Stiffness Matrix & Temp:- Force vector For the Plane stress for Element stress in fig. The Element experiences 20°C, increase in Temp. Assume coeff of thermal expansion is $6\times10^{-6}/^\circ C$ Take $E = 2\times10^5 \ N/mm^2$ & $\gamma = 0.25$ thickness = 5cm = 5×10^{-2} m.

Sol:-



Step 1 :-

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}.$$

$$= \frac{1}{2}\,(x_2 y_3 - x_3 y_2) - x_1 (y_3 - y_2) + y_1 (x_3 - x_2)$$

$$= \frac{1}{2}\,((2)(3) - (1)(0)) - 0 + 0\,(1-2)$$

$$= \frac{1}{2}\,(6)\,([illegible]) = 3 \ m^2.$$

Step2:- strain displacement Matrix [B].

$$= \frac{1}{2A}\begin{vmatrix} \alpha_1 & 0 & \alpha_2 & 0 & \alpha_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \alpha_1 & \gamma_2 & \alpha_2 & \gamma_3 & \alpha_3 \end{vmatrix}.$$

$\alpha_1 = Y_2 - Y_3 = 0 - 3 = -3$

$\alpha_2 = Y_3 - Y_1 = 3 - 0 = 3$

$\alpha_3 = Y_1 - Y_2 = 0 - 0 = 0$

$\gamma_1 = x_3 - x_2 = 1 - 2 = -1$

$\gamma_2 = x_1 - x_3 = 0 - 1 = -1$

$\gamma_3 = x_2 - x_1 = 2 - 0 = 2$.

$$= \frac{1}{2(3)} \begin{vmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{vmatrix}.$$

$$[B] = \frac{1}{6} \begin{vmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1. & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{vmatrix}.$$

$= 0.167$.

Step 3:-

Stress - Strain Relationship,

$$[D] = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1-\frac{v}{2} \end{bmatrix}$$

$$= \frac{2\times10^5}{1-0.25^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 1-\frac{0.25}{2} \end{bmatrix}$$

$$= 2.13\times10^{3} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 1-\frac{0.25}{2} \end{bmatrix}$$

$$= 53.330\times10^{3} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

Step 4:- $[D]\ [B]$.

$$53.33\times10^{3} \times 0.167 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -7 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}.$$

$$= 8852.78 \begin{bmatrix} -12+0+0 & 0-1+0 & 12+0+0 & 0-1+0 & 0 & 0+2+0 \\ 0-2+0 & 0-4+0 & 3+0+0 & 0-4+0 & 0+0+0 & 0+8 \\ 0+0-1.5 & 0+0-4.5 & -1.5 & 4.5 & 3.0 & 0 \end{bmatrix}$$

$$= 8852.78 \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -2 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

Step 5:- $[B^T]$.

$$= 0.167 \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Step 6:-

$$[k] = [B^T][D][B] \cdot A \cdot t$$

$$= 0.166 \times 53.33 \times 10^3 \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$= 8852.78 \times 10^3 \begin{bmatrix} -12+0+0 & -3+0+0 & 0+0-1.5 \\ 0-1+0 & 0-4+0 & 0+0-4.5 \\ 12+0+0 & 3+0+0 & 0+0-1.5 \\ 0-1+0 & 0-4+0 & 0+0+4.5 \\ 0+0+0 & 0+0+0 & 0+0+3.0 \\ 0+2+0 & 0+8+0 & 0+0+0 \end{bmatrix}$$

$$= 8852.78 \begin{bmatrix} -12 & -3 & -1.5 \\ -1 & -4 & -4.5 \\ 12 & 3 & -1.5 \\ -1 & -4 & 4.5 \\ 0 & 0 & 3.0 \\ 2 & 8 & 0 \end{bmatrix}.$$

$$= 8.88 \times 10^3$$

Step 7 $= [B^T] \; [D] \; [B]$.

$$= 8852.78 \times 0.167 \times 10^{-3} \begin{bmatrix} -2 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

$$= 1478.41 \begin{bmatrix} 36+0+1.5 & 3+0+4.5 & -36+0+1.5 & 3+0-4.5 & 0+0-3 & -6+0+0 \\ 0+3+4.5 & 0+4+13.5 & 0-3+4.5 & 0+4-13.5 & -9 & -8 \\ -36+1.5 & -3+4.5 & 36+1.5 & -3-4.5 & -3 & 6 \\ -3-4.5 & 4-13.5 & -3-4.5 & 4+13.5 & 9 & -8 \\ -2.5 & -9.0 & -3.0 & 5.0 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

$$= 1478.41 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -7.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -2.5 & -9 & -3 & 5 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

Step 8 :- $[K] = [B^T]\ [D]\ [B] \times A \times t$.

$$= 1478.41 \times 3 \times 5 \text{ in m}$$

$$\begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -7.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -2.5 & -9 & -3 & 5 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

$$[K] = 22176.15 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -7.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -2.5 & -9 & -3 & 5 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

$$= 22.176 \times 10^3 .$$

Step 9 :- Initial strain $(\varepsilon_0) = \begin{bmatrix} \alpha \cdot \Delta T \\ \alpha \cdot \Delta T \\ 0 \end{bmatrix}$

$$\varepsilon_0 = \begin{bmatrix} 6 \times 10^{-6} \times 20 \\ 6 \times 10^{-6} \times 20 \\ 0 \end{bmatrix}$$

$$\varepsilon_0 = \begin{bmatrix} 12 \times 10^{-5} \\ 12 \times 10^{-5} \\ 0 \end{bmatrix}$$

Step 10:- Temp Force vector [F] 5/11/2

$$= [B^T]\,[D]\,[e_0]\cdot A \times t\,.$$

$$[F] = 8.88\text{[illegible]} \times 10^{3} \begin{bmatrix} -12 & -3 & -1.5 \\ -1 & -4 & -4.5 \\ 12 & 3 & -1.5 \\ -1 & -4 & 4.5 \\ 0 & 0 & 3 \\ 2 & 8 & 0 \end{bmatrix} \begin{bmatrix} 12\times10^{5} \\ 12\times10^{5} \\ 0 \end{bmatrix}$$

$$= 8.88\text{[illegible]}028 \times 3 \times 5\times10^{-2}\times10^{5} \begin{bmatrix} -144-36+0 \\ -12-48+0 \\ 144+36+0 \\ -12-48 \\ 0 \\ 24+96 \end{bmatrix}$$

$$= 133.2\times10^{2} \times \begin{bmatrix} -180 \\ -60 \\ 180 \\ -58 \\ 0 \\ 110 \end{bmatrix} \div 2\,.$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} = \begin{bmatrix} -120 \\ -40 \\ 120 \\ 40 \\ 0 \\ 80 \end{bmatrix}$$

— × —

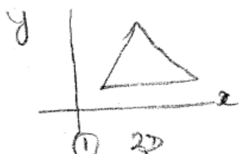
5/11/14

UNIT-IV.

4. For the Element square in the fig. Determine stiffness Matrix Take E = 200 Gpa and Poisson's ratio ν = 0.25, the co-ordinates are in mm.

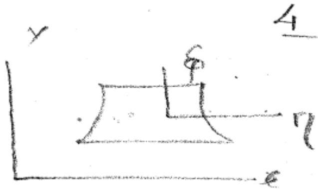
4 CASES

①



① 2D
↓
② St-F
③ Temp

②



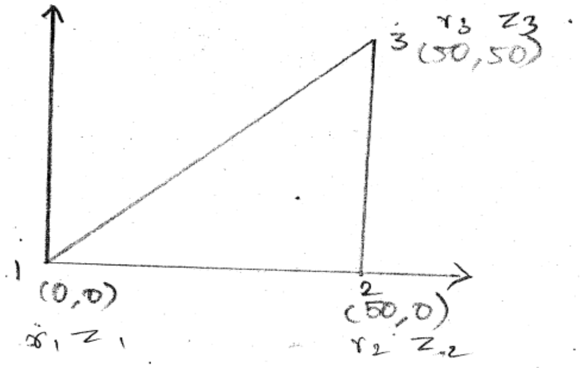
x
y
N1
N2

③ Numerical integration

④

2D
↓
Structural
↓
Temp

— x —



Q:-

For Axis symmetric problem that is $\triangle^{lular}$ Element the stiffness Matrix K is Given by $[K] = 2\pi r A [B]^T [D] [B]$

Step 1:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & r_1 & z_1 \\ 1 & r_2 & z_2 \\ 1 & r_3 & z_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 50 & 0 \\ 1 & 50 & 50 \end{vmatrix}$$

$$= 1250\,mm^2.$$

$$r = \frac{r_1 + r_2 + r_3}{3} = 33.33mm$$

$$z = \frac{z_1 + z_2 + z_3}{3} = 16.66mm.$$

Step 2:- Stress and Strain relationship Matrix,

$$[D] = \frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & v & 0 \\ v & 1-v & v & 0 \\ v & v & 1-v & 0 \\ 0 & 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$$

$$= \frac{200\times10^3 \,(N/mm^2)}{(1+0.25)(1-2(0.25))}\begin{bmatrix} 1-0.25 & 0.25 & 0.25 & 0 \\ 0.25 & 1-0.25 & 0.25 & 0 \\ 0.25 & 0.25 & 1-0.25 & 0 \\ 0 & 0 & 0 & \frac{1-2(0.25)}{2} \end{bmatrix}$$

$$= 80\times10^3\begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-III :-

$$[B] = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ \frac{\alpha_1}{r}+\beta_1+\frac{\gamma_1 z}{r} & 0 & \frac{\alpha_2}{r}+\beta_2+\frac{\gamma_2 z}{r} & 0 & \frac{\alpha_3}{r}+\beta_3+\frac{\gamma_3 z}{r} & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

$\alpha_1 = r_2 z_3 - r_3 z_2 = 2500\,mm$ $\qquad \gamma_1 = r_3 - r_2 = 0$

$\alpha_2 = r_3 z_1 - r_1 z_3 = 0$ $\qquad \gamma_2 = r_1 - r_3 = -50$

$\alpha_3 = r_1 z_2 - r_2 z_1 = 0$ $\qquad \gamma_3 = r_2 - r_1 = 50.$

$\beta_1 = z_2 - z_3 = -50$

$\beta_2 = z_3 - z_1 = 50$

$\beta_3 = z_1 - z_2 = 0$

$$[B] = \frac{1}{2(1250)} \begin{bmatrix} -50 & 0 & 50 & 0 & 0 & 0 \\ 25 & 0 & 25 & 0 & 25 & 0 \\ 0 & 0 & 0 & -50 & 0 & 50 \\ 0 & -50 & -50 & 50 & 50 & 0 \end{bmatrix}$$

$= 4\times10^{-4} \times 25$

step IV :- $\qquad [D] \quad [B]$

$$= 800 \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 2 \\ 0 & -2 & -2 & 2 & 2 & 0 \end{bmatrix}$$

$$= 800 \begin{bmatrix} -6+1 & 0 & 6+1 & -2 & 1 & 2 \\ -2+3 & 0 & 2+3 & -2 & 3 & 2 \\ -2+1 & 0 & 2+1 & -6 & 1 & 6 \\ 0 & -2 & -2 & 2 & 2 & 2 \end{bmatrix}$$

5.

$$\Rightarrow 800 \begin{bmatrix} -5 & 0 & 7 & -2 & 1 & 2 \\ -1 & 0 & 5 & -2 & 3 & 2 \\ -1 & 0 & 3 & -6 & 1 & 6 \\ 0 & -2 & -2 & 2 & 2 & 2 \end{bmatrix}$$

Step V :-

$$[B^T] = 0.01 \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 2 & 1 & 0 & -2 \\ 0 & 0 & -2 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

Step VI :-

$$[B^T] \ [D] \ [B].$$

$$= 8 \begin{bmatrix} 11 & 0 & -9 & 2 & 1 & -2 \\ 0 & 4 & 4 & -4 & -4 & 0 \\ -9 & 4 & 23 & -10 & 1 & 6 \\ 2 & -4 & -10 & 16 & 2 & 2 \\ 1 & -4 & 1 & 2 & 7 & \\ -2 & 0 & 6 & -12 & 2 & 12 \end{bmatrix}$$

Step VII :-

$$[K] = 2\pi r A \ [B^T] [D] [B].$$

$$= 2.094 \times 10^6 \begin{bmatrix} 11 & 0 & -9 & 2 & 1 & -2 \\ 0 & 4 & 4 & -4 & -4 & 0 \\ -9 & 4 & 23 & -10 & 1 & 6 \\ 2 & -4 & -10 & 16 & 2 & 2 \\ 1 & -4 & 1 & 2 & 7 & 2 \\ -2 & 0 & 6 & -12 & 2 & 12 \end{bmatrix} //$$

5. For the Axis symmetric Element shown in fig. Determine Element stresses, E = 210 Gpa, $\nu = 0.25$, the Nodal displacements are $u = 0.05mm$, $u_2 = 0.02$ mm, $u_3 = 0$ mm. $w_1 = 0.03mm$, $w_2 = 0.02$ mm, $w_3 = 0$.

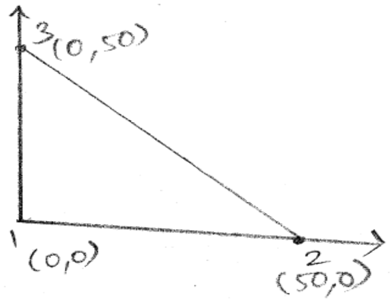
To find:-

$\sigma_r = ?$

$\sigma_\theta = ?$

$\sigma_z = ?$

$\tau_{rz} = ?$



S:-

W.K.T, stress is equivalent to,

$$\{\sigma\} = [D] \cdot [B] \cdot \{u\}.$$

$$= 1680 \begin{bmatrix} -2 & -1 & 4 & 0 & 1 & 1 \\ 2 & -1 & 4 & 0 & 3 & 1 \\ 0 & -3 & 2 & 0 & 1 & 3 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \end{bmatrix}$$

$$= 1680 \begin{bmatrix} -2 & -1 & 4 & 0 & 1 & 1 \\ 2 & -1 & 4 & 0 & 3 & 1 \\ 0 & -3 & 2 & 0 & 1 & 3 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.03 \\ 0.02 \\ 0.02 \\ 0 \\ 0 \end{bmatrix}.$$

$$[\sigma] = 1680 \begin{bmatrix} -0.1 & -0.03 & 0.08 & 0 & 0 & 0 \\ 0.1 & -0.03 & 0.08 & 0 & 0 & 0 \\ 0 & -0.09 & 0.04 & 0 & 0 & 0 \\ -0.05 & -0.03 & 0 & 0.02 & 0 & 0 \end{bmatrix}$$

$$[\sigma] = 1680 \begin{bmatrix} -0.05 \\ 0.15 \\ -0.05 \\ -0.06 \end{bmatrix}.$$

$\sigma_r = -84$ N/mm^2.

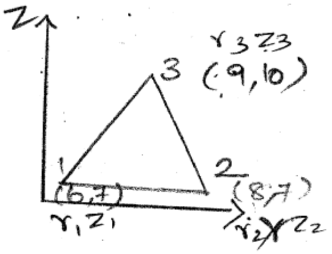
$\sigma_\theta = 252$ N/mm^2.

$\sigma_z = -84$ N/mm^2.

$\tau_{rz} = -100.8$ N/mm^2.

6/11/14

6) Calculate Element Stiffness Matrix and thermal Force vector For the Axis symmetric Triangular Element as shown in fig. The Element Experiences a 15°c increasing Temperature.



G:-

$$\alpha = 10\times10^{-6}/°C$$

$$E = 2\times10^{5}\ N/mm^2$$

$$\nu = 0.25$$

$$\Delta T = 15°C$$

S:- The Stiffness Matrix,

$$[K] = 2\pi r A \cdot [B^T]\cdot[D]\cdot[B].$$

Step 1:-

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & r_1 & z_1 \\ 1 & r_2 & z_2 \\ 1 & r_3 & z_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 6 & 7 \\ 1 & 8 & 7 \\ 1 & 9 & 10 \end{vmatrix}.$$

$$= \frac{1}{2}\ 1(80-63) - 6(10-7) + 7(9-8)$$

$$= \frac{1}{2}\ (17) - (18) + (7)$$

$$= 3\ mm^2.$$

$$r = r_1+r_2+r_3/_3 = \frac{6+8+9}{3} = \frac{23}{3}$$

$$r = 7.66\ mm.$$

$$z = \frac{z_1+z_2+z_3}{3} = \frac{7+7+10}{3} = 8\,mm.$$

Step 2 :-

$$[B] = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ \frac{\alpha_1}{r}+\beta_1+\frac{\gamma_1 z}{r} & 0 & \frac{\alpha_2}{r}+\beta_2+\frac{\gamma_2 z}{r} & 0 & \frac{\alpha_3}{r}+\beta_3+\frac{\gamma_3 z}{r} & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

$\alpha_1 = r_2 z_3 - r_3 z_2 = (8)(10) - (9)(7) = 17\,mm^2$ $\quad \gamma_1 = r_3 - r_2 = 1$

$\alpha_2 = r_3 z_1 - r_1 z_3 = (9)(7) - (6)(10) = 63 - 60 = 3\,mm^2$ $\quad \gamma_2 = r_1 - r_3 = -3$

$\alpha_3 = r_1 z_2 - r_2 z_1 = (6)(7) - (8)(7) = 42 - 56 = -14$ $\quad \gamma_3 = r_2 - r_1 = -2.$

$\beta_1 = z_2 - z_3 = -3$

$\beta_2 = z_3 - z_1 = 3$

$\beta_3 = z_1 - z_2 = 0$

$$= \frac{1}{6}\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ \frac{17}{7.66}+(-3)+\frac{(1)(8)}{7.66} & 0 & \frac{3}{7.66}+3+\frac{(-3)(8)}{7.66} & 0 & \frac{-14}{7.66}+0+\frac{2(8)}{7.66} & 0 \\ 0 & 1 & 0 & -3 & 0 & 2 \\ 1 & -3 & -3 & 3 & 2 & 0 \end{bmatrix}$$

$$= 0.167\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ +0.26 & 0 & 0.25 & 0 & 0.26 & 0 \\ 0 & 1 & 0 & -3 & 0 & 2 \\ 1 & -3 & -3 & 3 & 2 & 0 \end{bmatrix}$$

Step 3 :-
$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$= \frac{2\times10^{5}}{(1+0.25)(1-2(0.25))} \begin{bmatrix} 1-0.25 & 0.25 & 0.25 & 0 \\ 0.25 & 1-0.25 & 0.25 & 0 \\ 0.25 & 0.25 & 1-0.25 & 0 \\ 0 & 0 & 0 & \frac{1-2(0.25)}{2} \end{bmatrix}$$

$$[D] = 80\times10^{3} \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 4 :- $[D]\ [B]$:-

$$= 80\times10^{3}\times0.167 \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0.26 & 0 & 0.25 & 0 & 0.26 & 0 \\ 0 & 4 & 0 & -2 & 0 & 2 \\ 1 & -3 & -3 & 3 & 2 & 0 \end{bmatrix}$$

$$= 13360 \begin{bmatrix} -9+0.26 & 1 & 9+0.25 & -2 & 0.26 & 2 \\ -3+.78 & 1 & 3+.75 & -2 & .78 & 2 \\ -3+0.26 & 3 & 3+0.25 & -4 & 0.26 & 6 \\ 1 & -3 & -3 & 3 & 2 & 0 \end{bmatrix}$$

$$[D][B] = 13360 \begin{bmatrix} -8.74 & 1 & 9.25 & -2 & 0.26 & 2 \\ -2.22 & 1 & 3.75 & -2 & 0.78 & 2 \\ -2.74 & 3 & 3.25 & -9 & 0.26 & 6 \\ 1 & -3 & -3 & 3 & 2 & 0 \end{bmatrix}.$$

Step V :-

$$[B^T] = 0.167 \begin{bmatrix} -3 & 0.26 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 3 & 0.25 & 0 & -3 \\ 0 & 0 & -3 & 3 \\ 0 & 0.26 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}.$$

Step VI :- $[K] = [B^T]\,[D]\,[B] \times 2\pi r A$.

$[B^T]\,[D]\,[B] =$

0.167×13360

$$\Rightarrow \begin{bmatrix} -3 & 0.26 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 3 & 0.25 & 0 & -3 \\ 0 & 0 & -2 & 3 \\ 0 & 0.26 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -8.74 & 1 & 9.25 & -2 & 0.26 & 2 \\ -2.22 & 1 & 3.75 & -2 & 0.78 & 2 \\ -2.74 & 3 & 3.25 & -9 & 0.26 & 6 \\ 1 & -3 & -3 & 3 & 2 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 26.22 - 0.57 + 1 \\ \\ \\ \\ \end{bmatrix}$$

$$[k] = [B^T]\,[D]\,[B] \times 2\pi r A$$

$$= 321.26\times10^{3}\begin{bmatrix} 26.3 & -5.7 & -29.7 & 11.21 & 1.42 & -5.4 \\ -5.7 & 12.0 & 12.2 & -18.0 & -5.7 & 6.0 \\ -29.7 & 12.26 & 37.7 & -18.7 & -5.01 & 6.52 \\ 11.21 & -18.0 & -18.7 & 36 & 5.21 & -18.0 \\ 1.42 & -5.73 & -5.01 & 5.21 & 4.20 & 0.52 \\ -5.47 & 6.0 & 6.52 & -18.0 & 0.52 & 12.0 \end{bmatrix}$$

Step - VII :- Thermal Force vector

$$[F] = [B^T]\,[D]\,\{e\}\,2\pi r A$$

Strain $\{e\}_t = \begin{Bmatrix} \alpha \cdot \Delta T \\ \alpha \cdot \Delta T \\ \alpha \cdot \Delta T \end{Bmatrix}$

$$\{e\}_t = \begin{bmatrix} 10\times10^{-6}\times15 \\ 10\times10^{-6}\times15 \\ 10\times10^{-6}\times15 \end{bmatrix} = \begin{bmatrix} 150 \\ 150 \\ 150 \end{bmatrix}\times10^{-6}$$

$$\Rightarrow [B^T]\,[D]$$

$$\Rightarrow 0.167 \times 80 \times 10^3 \begin{bmatrix} -3 & 0.26 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 3 & 0.25 & 0 & -3 \\ 0 & 0 & -3 & 3 \\ 0 & 0.26 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\Rightarrow 13360 \begin{bmatrix} -9+0.26 & -3+.78 & -3+0.26 & 1 \\ 1 & 1 & 3 & -3 \\ 9+0.25 & 3+.75 & 3+0.25 & -3 \\ -3 & -3 & -9 & 3 \\ 0.26 & 0.78 & 0.26 & 2 \\ 2 & 2 & 6 & 0 \end{bmatrix}.$$

$$\Rightarrow 13360 \begin{bmatrix} -8.74 & -2.22 & -2.74 & 1 \\ 1 & 1 & 3 & -3 \\ 9.25 & 3.75 & 3.25 & -3 \\ -3 & -3 & -9 & 3 \\ 0.26 & 0.78 & 0.26 & 2 \\ 2 & 2 & 6 & 0 \end{bmatrix}$$

$$[F]_t = [B^T]\ [D]\ [e]\ 2\pi r A.$$

$$\Rightarrow 13360 \times 2\pi \times 3 \times 7.66 \times 10^{-6}.$$

$$\begin{bmatrix} -8.74 & -2.22 & & \\ & & & 1 \\ 1 & 3 & -2.74 & -3 \\ 9.25 & 3.75 & 3 & -3 \\ -3 & -3 & 3.25 & 3 \\ 0.26 & 0.78 & -9 & 2 \\ 2 & & 0.26 & \\ & 2 & -6 & 0 \end{bmatrix}^{T} \begin{bmatrix} 150 \\ 150 \\ 0 \\ 150 \end{bmatrix}$$

$$\Rightarrow. \begin{bmatrix} -1311 - 333 - 411 + 150. \\ 150 + 450 + 0 - 450 \\ 1387.5 + 562.5 + 0 + 450 \\ -450 - 450 + 0 + 300 \\ 39 + 117 + 0 + 300 \\ 300 + 300 + 0 + 0 \end{bmatrix}.$$

$$\begin{bmatrix} F_1 u \\ F_1 w \\ F_2 u \\ F_2 w \\ F_3 w \\ F_3 u \\ F_3 w \end{bmatrix} \Rightarrow. \; 1 \begin{bmatrix} -2878.25 \\ -289.08 \\ 2903.45 \\ -867.25 \\ 879.86 \\ 1156.34 \end{bmatrix} N.$$

UNIT-I

1. Stress strain Relationship.
2. Explain Hook's law.
3. Explain briefly Temp effect.
4. Write short Notes on Saint venants prin. (4M or 8M)
5. Explain Galerikan's Method with Example.
6. " Rayleigh's Ritz Method with Ex
7. ADV and Disadv of FEM.
8. Procedure of FEM.
9. Explain Numbering and Nodes.
10. Explain sources of Error.

UNIT-II.

1. Explain Properties of Matrices and Determinants.
2. Wat is Cholesky Factorisation.
3. Explain conjugate gradient Method
4. Wat is skyline storage banded Matrix.

UNIT-III

1. Wat is shape Function.
2. Explain Principle of Minimum potential Energy.
3. Explain Properties of stiffness Matrix. (symmetrical)
4. Explain or solve derivation of stiffness Matrix for 1-D linear bar Element.
5. Explain Global, Local and Natural co ordinates.
6. Explain Natural and Essential boundary condition.
7. Wat is the diff b/n boundary Value Pbm & initial value pbms.

UNIT-IV

1) Wat is CST, LST, QST Element.

2) wat is the purpose of isoparametric Element.

3) Write down the shape functions for Four Noded Triangular Element Using Natural co-ordinate system.

4) D/b/n. super parametric, sub parametric, iso-Parametric element.

5) Wat is Axis symmetric Element.

6. Wat are the types of Non-linearity

—x—

1) A Furnance Wall is Made Up of three layers, inside layer with thermal conductivity 8.5 W/mk, Middle layer with cvy 0.25 W/mk. the outer layer with cvy 0.08 W/mk. The Respective Thickness of Inner, Middle and Outer layer are 25cm, 5cm, 3cm resly. The inside temp of wall is 600°c and Outside of wall is exposed to atm. air at 30°c with Heat Transfer coeff of 45 W/m²k. Determine the Nodal Temperatures.

G :- $K_1 = 8.5 \text{ W/mK}$, $K_2 = 0.25 \text{ W/mK}$.

$K_3 = 0.08 \text{ W/mK}$, $l_1 = 25\text{cm} = 0.25\text{m}$, $l_2 = 5\text{cm} = 0.05\text{m}$

$l_3 = 3\text{cm} = 0.03\text{m}$.

$T_1 = 600°c + 273°c = 873 \text{ K}$.

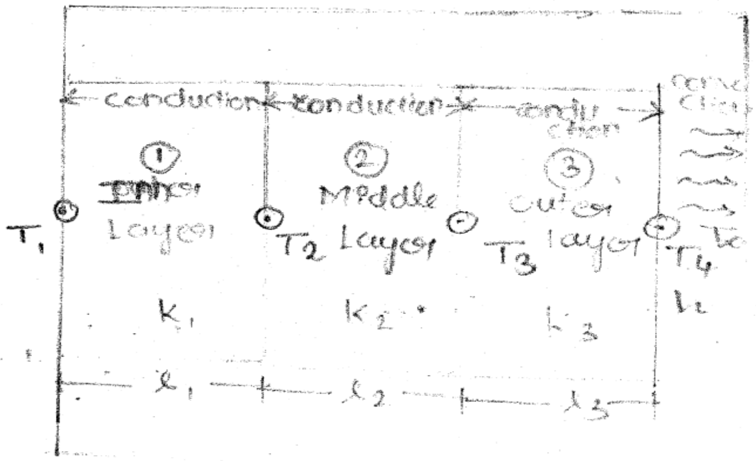
$T_\infty = 30°c + 273°c = 303 \text{ K}$.

$h = 45 \text{ W/m}^2\text{K}$.

To find :-

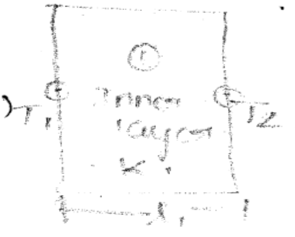
Nodal Temperatures, T_2, T_3, T_4 = ?

Sol :-



i). Nodal (1, 2) :-

$$\frac{A_1 K_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}T_1\\ T_2\end{bmatrix} = \begin{bmatrix}F_1\\ F_2\end{bmatrix} \longrightarrow (1)$$

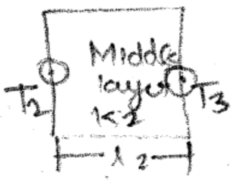


Assume $A_1 = 1\text{m}^2$ in (1)

$$\frac{1 \times 8.5}{0.25}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}T_1\\ T_2\end{bmatrix} = \begin{bmatrix}F_1\\ F_2\end{bmatrix}$$

$$\begin{array}{c|cc|} & 1 & 2 & \\ \hline & \begin{bmatrix}34 & -34\\ -34 & 34\end{bmatrix} & & \begin{matrix}1\\2\end{matrix}\end{array}\begin{bmatrix}T_1\\ T_2\end{bmatrix} = \begin{bmatrix}F_1\\ F_2\end{bmatrix} \longrightarrow (2).$$

ii) Nodal (2,3):-



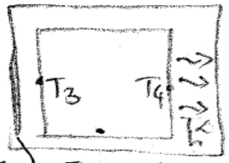
$$\frac{A_2 k_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}T_2\\ T_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}.$$

$$\frac{0.25}{0.05}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}T_2\\ T_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}$$

$$\begin{array}{cc} 2 & 3 & \\ \end{array}$$

$$\begin{array}{c|cc|c} & 2 & 3 & \\ \hline & 5 & -5 & 2\\ & -5 & 5 & 3 \end{array}\begin{bmatrix}T_2\\ T_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}.$$

iii). Nodal (3, 4):-



$$\left(\frac{A_3 k_3}{l_3}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+hA\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right)\begin{bmatrix}T_3\\ T_4\end{bmatrix}$$

$$=hT_\infty A\begin{bmatrix}0\\ 1\end{bmatrix}.$$

$$\Rightarrow\left(\frac{1\times 0.08}{0.03}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+45\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right)\begin{bmatrix}T_3\\ T_4\end{bmatrix}$$

$$=45\times 303\begin{bmatrix}0\\ 1\end{bmatrix}.$$

$$\left(\begin{bmatrix}2.66 & -2.66\\ -2.66 & 2.66\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 45\end{bmatrix}\right)\begin{bmatrix}T_3\\ T_4\end{bmatrix}$$

$$=\begin{bmatrix}0\\ 13.635\times 10^3\end{bmatrix}.$$

$$\begin{array}{cc|c} 3 & 4 & \\ \hline 2.66 & -2.66 & 3\\ -2.66 & 47.66 & 4 \end{array}\begin{bmatrix}T_3\\ T_4\end{bmatrix}=\begin{bmatrix}0\\ 13.635\times 10^3\end{bmatrix}.$$

Step 1:- Assemble the Finite element eqn (1), (2) & (3)

$$\begin{bmatrix} 34 & -34 & 0 & 0 \\ -34 & 34+5 & -5 & 0 \\ 0 & -5 & 5+2.66 & -2.66 \\ 0 & 0 & -2.66 & 47.66 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 34 & -34 & 0 & 0 \\ -34 & 39 & -5 & 0 \\ 0 & -5 & 7.666 & -2.66 \\ 0 & 0 & -2.66 & 47.66 \end{bmatrix}}_{\{K\}} \underbrace{\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}}_{\{T\}} = \underbrace{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{\{F\}} \rightarrow (4)$$

$$(F_1) = (F_2) = (F_3) = 0$$

$$(F_4) = 13.635 \times 10^3 \text{ in } (4)$$

$$\begin{bmatrix} 34 & -34 & 0 & 0 \\ -34 & 39 & -5 & 0 \\ 0 & -5 & 7.666 & -2.66 \\ 0 & 0 & -2.66 & 47.66 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 13.635 \times 10^3 \end{bmatrix} \longrightarrow (5)$$

Step 2 :- The First row and First column Matrix is set equal to 0 .

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 39 & -5 & 0 \\ 0 & -5 & 7.666 & -2.66 \\ 0 & 0 & -2.66 & 47.66 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 13.635\times10^3 \end{bmatrix}$$

Step 3 :-

First row of Force Matrix is replaced by known Temp T_1

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 39 & -5 & 0 \\ 0 & -5 & 7.666 & 2.66 \\ 0 & 0 & -2.66 & 47.66 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 873 \\ 0 \\ 0 \\ 13.635\times10^3 \end{bmatrix}$$

Step 4 :-

The second row first value of Stiffness Matrix is Multiplied by known Temp T_1, -34×873 is added to Positive second Force vector,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 39 & -5 & 0 \\ 0 & -5 & 7.666 & -2.66 \\ 0 & 0 & -2.66 & 47.66 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 873 \\ 29682 \\ 0 \\ 13.635\times10^3 \end{bmatrix}$$

↳ (6) .

Step 5 :- . By Gauss elimination Method . We can find the solution –

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 873 \\ 0 & 39 & -5 & 0 & 29682 \\ 0 & -5 & 7.666 & -2.66 & 0 \\ 0 & 0 & -2.66 & 47.66 & 13.635 \times 10^3 \end{bmatrix} \quad R_2 \rightarrow R_2 \times 1/39$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 873 \\ 0 & 1 & -0.128 & 0 & 761.076 \\ 0 & -5 & 7.666 & -2.66 & 0 \\ 0 & 0 & -2.66 & 47.66 & 13.635 \times 10^3 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + 5R_2 .$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 873 \\ 0 & 1 & -0.128 & 0 & 761.076 \\ 0 & 0 & 7.026 & -2.666 & 3805.38 \\ 0 & 0 & -2.666 & 47.66 & 13.635 \times 10^3 \end{bmatrix}$$

$$R_3 \rightarrow R_3 \times \frac{1}{7.026} .$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 873 \\ 0 & 1 & -0.128 & 0 & 761.076 \\ 0 & 0 & 1 & -0.379 & 541.614 \\ 0 & 0 & -2.666 & 47.666 & 13.635 \times 10^3 \end{bmatrix} .$$

$$R_4 \rightarrow R_4 + 2.666 R_3 .$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 873 \\ 0 & 1 & -0.128 & 0 & 761.076 \\ 0 & 0 & 1 & -0.379 & 541.614 \\ 0 & 0 & 0 & 46.655 & 15.079 \times 10^3 \end{bmatrix} .$$

$T_4 = 323.21K$, $T_3 = 664.11K$, $T_2 = 84608K$.

1/8/14.

THEORY - UNIT-I.

1. Saint Venant's Principle:-

* We often Have to make approximations in defining boundary conditions to represent a support - structure interface.

* For instance consider a Cantilever beam, free at one end and attached to a column with rivets at the other end.

* Questions arise as to whether the riveted Joints is totally rigid or partially rigid, and as to whether each point on the cross section at the fixed end is specified to have the same boundary conditions.

* Saint Venant considered the effect of different approximations on the solution to the total problem.

* Saint Venant's Principle states that, as long as the different approximations are statically equivalent, the resulting solutions will be valid provided we focus on regions sufficiently far away from the support. That is, the solutions may significantly differ only within the immediate vicinity of the support.

2. Rayleigh - Ritz Method :-

* In applied Mathematics and Mechanical engineering, the Rayleigh - Ritz Method (after walther Ritz and Lord Rayleigh) is a widely used, classical Method for the calculation of Natural vibration Frequency of a structure in the second or Higher order.

* It is a direct Variational Method in which the Minimum of a functional defined on a normal linear space is approximated by a linear combination of elements from that space.

* This Method will yield solutions when an analytical Form for the true solution may be intractable.

* The Method is also widely used in quantum chemistry.

* Typically in Mechanical Engineering it is used For Finding the approximate real resonant Frequencies of Multi degree systems, such as spring Mass systems or Flywheels on a shaft with varying cross section.

* It can also be used for Finding buckling loads and Post - buckling behaviour for columns.

* Total potential energy of the structure is given by:-

$$\pi = \left\{ \begin{array}{c} \text{Internal} \\ \text{Potential} \\ \text{Energy} \end{array} \right\} - \left\{ \begin{array}{c} \text{External} \\ \text{Potential} \\ \text{Energy} \end{array} \right\}.$$

$$\pi = U - H \cdot$$

* In this Method for Continuous System we deal with the fall functional potential energy,

$$\pi = \int_{x_1}^{x_2} f(y, y', y'') dx \cdot$$

— × —

3) GALERKIN'S METHOD :-

* Galerkin's Method uses the set of governing equations in the developement of an integral form.

* It is usually presented as one of the weighted residual Methods.

* Galerkin's Method works direct from the differential equation and is preferred to the Rayleigh-Ritz Method for problems where a corresponding function to be minimized is not obtainable.

* In Mathematics, in the area of Numerical Analysis, galerkin Methods are a class of Methods for converting a Continuous operator problem (such as differential equation) to a discrete problem.

* In principle, It is the equivalent of applying the Method of variation of parameters to a function space, by converting the ean to a weak formulation.

* Typically one then applies some constraints on the Function space to characterise the space with a finite set of basic functions

* The Approach is Usually credited to the Russian Mathematician Galerkin but the Method was discovered by the swiss Mathematician Walther Ritz ;(1) to whom Galerkin refers.

— x —

UNIT-II.

(1) CONJUGATE GRADIENT METHOD:-

* the conjugate Gradient Method is an iterative Method for the solution of equations.

* This Method is becoming increasingly Popular and is implemented in several computer codes. ~~we~~ The Fletcher - Reeves version of algorithm For symmetric Matrices.

* consider the solution of set of equations,

$$Ax = b,$$

* Where A is a symmetric Positive definite (n×n), Matrix, ~~b~~ and x are (n×1). The Conjugate Gradient Method Uses the following steps for symmetric A.

8) GAUSS ELIMINATION METHOD:- In this Method the given system is transformed system with upper - triangular coefficient Matrix which can be solved by back substitution.

Ex:- $10x - 2y + 3z = 23$,

$2x + 10y - 5z = -33$.

$3x - 4y + 10z = 41$.

STEP 1:-

$$\begin{pmatrix} 10 & -2 & 3 & 23 \\ 2 & 10 & -5 & -33 \\ 3 & -4 & 10 & 41 \end{pmatrix}$$

STEP 2:-

$$R_1 \to R_1 \div 10 \begin{pmatrix} 1 & -1/5 & 3/10 & 23/10 \\ 2 & 10 & -5 & -33 \\ 3 & -4 & 10 & 41 \end{pmatrix}$$

$$R_2 \to R_2 - 2R_1 , \ R_3 \to R_3 - 3R_1$$

STEP 3:-

$$\begin{pmatrix} 1 & -1/5 & 3/10 & 23/10 \\ 0 & 52/5 & -28/5 & -188/5 \\ 0 & -17/5 & 91/10 & 341/10 \end{pmatrix}$$

STEP 4:-

$$R_2 \to R_2 \div 52/5$$

$$\begin{pmatrix} 1 & \frac{-1}{5} & \frac{3}{10} & \frac{23}{10} \\ 0 & 1 & \frac{-7}{13} & \frac{-47}{13} \\ 0 & \frac{-17}{5} & \frac{91}{10} & \frac{341}{10} \end{pmatrix}.$$

Step 5:- $K_3 \rightarrow K_3 + (\frac{1}{3})K_2$

$$\begin{pmatrix} 1 & \frac{-1}{5} & 3/10 & 23/10 \\ 0 & 1 & -7/13 & -47/13 \\ 0 & 0 & 189/26 & 567/26 \end{pmatrix}$$

Step 6:- $R_3 \rightarrow R_3 \div 189/26$

$$\begin{pmatrix} 1 & \frac{-1}{5} & \frac{3}{10} & \frac{23}{10} \\ 0 & 1 & \frac{-7}{13} & \frac{-47}{13} \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

Step 7:-

$$\boxed{\therefore Z = 3}$$

$$y + \left(\frac{-7}{13}\right) = -\frac{47}{13}$$

$$\Rightarrow y = -2$$

$$x - \frac{1}{5}y + \frac{3}{10}z = \frac{23}{10}$$

$$\Rightarrow 10x - 2y + 3z = 23$$

$$\Rightarrow 10x - 2(-2) + 3 \times 3 = 23$$

$$\Rightarrow 10x + 4 + 9 + 23$$

$$\boxed{x = 1.}$$

— × —

3) **Gauss Jordan Method:-** In this method the coefficient Matrix is reduced to an unit Matrix and directly we can find the unknowns,

Ex:-

$$x + 3y + 3z = 16,$$
$$x + 4y + 3z = 18,$$
$$x + 3y + 4z = 19$$

Step 1:-

$$\begin{pmatrix} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{pmatrix}$$

Step 2:-

$$R_2 \rightarrow R_2 - R_1$$
$$R_3 \rightarrow R_3 - R_1$$

$$\begin{pmatrix} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

Step 3:-

$$\begin{pmatrix} 1 & 0 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} \quad R_1 \rightarrow R_1 - 3R_2$$

Step 4:-

$$R_1 \rightarrow R_1 - 3R_3$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

The Matrix finally reduces to the form

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Therefore $\boxed{x=1, y=2, z=3}$.

$\therefore$ checking :- $x+3y+3z = 16$.

$$1+3(2)+3(3) = 16$$

$$1+6+9 = 16$$

$$\boxed{16 = 16}$$

—x—

4) **GAUSS SEIDAL METHOD:-**

In this Method the latest values of Unknowns at each stage of iteration are used in proceeding to the Next stage of iteration.

EG:- $4x+2y+z = 14, \longrightarrow (1)$

$x+5y-z = 10, \longrightarrow (2)$

$x+y+8z = 20 \longrightarrow (3)$.

$\therefore x = 1/4\ (14-2y-z) \longrightarrow (4)$

$y = 1/5\ (10-x+z) \longrightarrow (5)$.

$z = 1/8\ (20-x-y) \longrightarrow (6)$.

1st iteration,

Putting $y=0$, $z=0$ in (4) we get, $x = 14/4 = 3.5$.

Putting $x=3.5$, $z=0$ in (5), we get,

$y = \frac{1}{5}[10 - 3.5 + 0] = 1.3$

Putting $x = 3.5$, $y = 1.3$ in (6) we get,

$z = \frac{1}{8}[20 - 3.5 - 1.3] = 1.9$

$\therefore x = 3.5$, $y = 1.3$, $z = 1.9$.

* Second . Iteration:-

putting $y = 1.3$, $z = 1.9$ in (4) we get

$x = \frac{1}{4}[14 - 2(1.3) - 1.9] = 2.375$

Putting $x = 2.375$, $z = 1.9$ in (5) we get.

$y = \frac{1}{5}[10 - 2.375 + 1.9] = 1.905$.

putting $x = 2.375$ $y = 1.905$ in (6) we get,

$z = \frac{1}{8}[20 - 2.375 - 1.905] = 1.965$.

* Third iteration:-

Putting $y = 1.905$, $z = 1.965$ in (4) we get,

$x = 1/4 [14 - 2(1.905) - 1.965] = 2.056$

Putting $x = 2.056$, $z = 1.965$ in (5) we get.

$y = \frac{1}{5}[10 - 2.056 + 1.965] = 1.982$

Putting $x = 2.056$, $y = 1.982$ in (6) we get,

$z = \frac{1}{8}[20 - 2.056 - 1.982] = 1.995$.

* Fourth iteration:-

Putting $y = 1.982$ $z = 1.995$ in ④ we get,

$x = \frac{1}{4}[14 - 2(1.982) - 1.995] = 2.010$.

Putting $x = 2.010$,

$$y = \frac{1}{5}[10 - 2.010 + 1.995] = 1.997$$

Putting $x = 2.010$, $y = 1.997$ in (6), we get,

$$z = \frac{1}{8}[20 - 2.010 - 1.997] = 1.999$$

$\therefore x = 2.010$, $y = 1.997$, $z = 1.999$.

* Fifth Iteration:-

Putting $y = 1.997$; $z = 1.999$ in (4) we get,

$$x = \frac{1}{4}[14 - 2(1.997) - 1.999]$$

$$x = 2.001$$

Putting $x = 2.001$, $z = 1.999$ in (5) we get,

$$y = \frac{1}{5}[10 - 2.001 + 1.999] = 1.999$$

Putting $x = 2.001$, $y = 1.999$ in (6) we get,

$$z = \frac{1}{8}[20 - 2.001 - 1.999] = 2$$

$\therefore x = 2.001$, $y = 1.999$, $z = 2$.

* Sixth iteration:-

Putting $y = 1.999$, $z = 2$ in (4) we get,

$$x = \frac{1}{4}[14 - 2(1.999) - 2] = 2.001 .$$

Putting $x = 2.001$, $z = 2$ in (5) we get,

$$y = \frac{1}{5}[10 - 2.001 + 2] = 1.999$$

Putting $x = 2.001$, $y = 1.999$ in (6) we get,

$$z = 1/8\,[20 - 2.001 - 1.999] = 2$$

$\therefore$ In the sixth iteration we get,

$x = 2.001$, $y = 1.999$, $z = 2$.

5) GAUSS JACOBI METHOD:-

$$10x-5y-2z=3.$$
$$4x-10y+3z=-3.$$
$$x+6y+10z=-3$$

S:- Here we see that the diagonal elements are dominants. Hence the iteration process can be applied.

That is the coefficient Matrix $\begin{pmatrix}10 & -5 & -2\\ 4 & -10 & 3\\ 1 & 6 & 10\end{pmatrix}$ is diagonally dominant since

$|10|>|-5|+|-2|$, $|-10|>|4|+|3|$ & $|10|>|1|+|6|$.

solving for x, y, z we have,

$$x=\frac{1}{10}(3+5y+2z)\longrightarrow(1)$$
$$y=\frac{1}{10}(3+4x+3z)\longrightarrow(2)$$
$$z=\frac{1}{10}(-3-x-6y)\longrightarrow(3)$$

* 1st Iteration:- Let the initial value be $(0,0,0)$, using these initial value is (1), (2), (3), we get,

$$x(1)=\frac{1}{10}[3+5(0)+2(0)]=0.3$$
$$y(1)=\frac{1}{10}[3+4(0)+3(0)]=0.3$$
$$z(1)=\frac{1}{10}[-3-(0)-6(0)]=0.3$$

* 2nd Iteration:-

Using these values in (1), (2), (3) we get

$x^{(2)} = \frac{1}{10}[3+5(0.3)+2(-0.3)] = 0.39$.

$y^{(2)} = \frac{1}{10}[3+4(0.3)+3(-0.3)] = 0.33$.

$z^{(2)} = \frac{1}{10}[-3-(0.3)-6(0.3)] = -0.51$.

* 3rd iteration:- using the values of $x^{(2)}, y^{(2)}, z^{(2)}$, in 1, 2, 3 we get,

$x^{(3)} = \frac{1}{10}[3+5(0.33)+2(-0.51)] = 0.363$

$y^{(3)} = \frac{1}{10}[3+4(0.39)+3(-0.51)] = 0.303$.

$z^{(3)} = \frac{1}{10}[-3-(0.39)-6(0.33)] = -0.537$.

* 4th iteration:-

$x^{(4)} = \frac{1}{10}[3+5(0.303)+2(-0.537)] = 0.3441$.

$y^{(4)} = \frac{1}{10}[3+4(0.363)+3(-0.537)] = 0.2841$.

$z^{(4)} = \frac{1}{10}[-3-0.363-6(0.3031)] = -0.5181$.

* 5th iteration:-

$x^{(5)} = \frac{1}{10}[3+5(0.2841)+2(-0.5181)] = 0.33843$.

$y^{(5)} = \frac{1}{10}[3+4(0.3441)+3(-0.5181)] = 0.2822$.

$z^{(5)} = \frac{1}{10}[-3-(0.3441)+6(0.2841)] = -0.50487$

* 6th iteration:-

$x^{(6)} = \frac{1}{10}[3+5(0.2822)+2(-0.50487)] = 0.340216$

$y^{(6)} = \frac{1}{10}[3+4(0.33843)+3(-0.50487)] = 0.283911$.

$z^{(6)} = \frac{1}{10}[-3-(0.33843)-6(0.2822)]$.

0.502162.

* 7th Iteration :-

$$x^{(7)} = \frac{1}{10}\left[3+5(0.283911)+2(-0.503163)\right]$$
$$= 0.3413229$$

$$y^{(7)} = \frac{1}{10}\left[3+4(0.340126)+3(-0.503163)\right]$$
$$= 0.2851015 \ .$$

$$z^{(7)} = \frac{1}{10}\left[-3-(0.340126)-6(0.283911)\right] = -0.5043592$$

* 8th Iteration :-

$$x^{(8)} = \frac{1}{10}\left[3+5(0.2851015)+2(-0.5043592)\right]$$
$$= 0.34167891$$

$$y^{(8)} = \frac{1}{10}\left[3+4(0.3413229)+3(-0.5043592)\right]$$
$$= 0.2852214$$

$$z^{(8)} = \frac{1}{10}\left[-3-(0.3413229)-6(0.2851015)\right]$$
$$= -0.50519319.$$

* 9th Iteration :-

$$x^{(9)} = \frac{1}{10}\left[3+5(0.2852214)+2(-0.50519319)\right]$$
$$= 0.341572062$$

$$y^{(9)} = \frac{1}{10}\left[3+4(0.34167891)+3(-0.50519319)\right]$$
$$= 0.285113607$$

$$z^{(9)} = \frac{1}{10}\left[-3-(0.34167891)-6(0.2852214)\right]$$
$$= 0.505300731 \ .$$

Hence correct to 3 decimal places, the values are,

$x = 0.342$, $y = 0.285$, $z = -0.505$.

6) EIGEN VALUE :- $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$

Let, the given Matrix be

$$A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$$

The characteristic equation is $|A - \lambda I| = 0$

(i,e) $$\begin{vmatrix} 2-\lambda & 2 & 0 \\ 2 & 1-\lambda & 1 \\ -7 & 2 & -3-\lambda \end{vmatrix} = 0$$

(i,e) $(2-\lambda)\,[(1-\lambda)(-3-\lambda)-2] - 2[2-(-3-\lambda)+7] = 0$

$(2-\lambda)\,[\lambda^2+2\lambda-5] - 2[1-2\lambda] = 0$

$2\lambda^2+4\lambda-10-\lambda^3-2\lambda^2+5\lambda-2+4\lambda = 0.$

$\lambda^3 - 13\lambda + 12 = 0$

solving the equation,

we get 3 values for λ

$\boxed{\lambda = 1, 3, -4}$

∴ Therefore the eigen Values are 1, 3, −4 ..

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7) EIGEN VECTOR :-

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$$

Let $X_1 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be the eigen Vector Corresponding to the eigen value $\lambda = 1$.

Then from the equation,

$(A - \lambda I)X_1 = 0$, we have,

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ -7 & 2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(i,e) $x_1 + 2x_2 + 0x_3 = 0$,

$2x_1 + 0x_2 + x_3 = 0$,

$-7x_1 + 2x_2 - 4x_3 = 0$.

Considering First two equations and using Gross rule Method, we have,

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ 2 & 0 & 1 & 2 \\ 0 & 1 & 2 & 0 \end{array}$$

$$\frac{x_1}{2-0} = \frac{x_2}{0-1} = \frac{x_3}{0-4} = k.$$

(i,e) $x_1 = 2k$,

$x_2 = -k$,

$x_3 = -4k$.

Hence the general eigenvector is

$X = \begin{pmatrix} 2k \\ -k \\ -4k \end{pmatrix}$.

Putting K=L, We get the simplest eigen vector

$$X_1 = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$$

Let, $X_2 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be the Eigen Vector Corresponding to $\lambda = 3$. Then the equation $(A-\lambda I)X_2 = 0$ becomes,

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & -2 & 1 \\ -7 & 2 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(i,e), $-x_1 + 2x_2 + 0x_3 = 0.$

$2x_1 - 2x_2 + x_3 = 0.$

$-7x_1 + 2x_2 - 6x_3 = 0$

Considering First Two Equations and applying rule of Cross Multiplication we have,

$$\begin{array}{ccccccc} x_1 & & & x_2 & & & x_3 \\ 2 & & 0 & & -1 & & 2 \\ -2 & & 1 & & 2 & & -2 \end{array}$$

$$\frac{x_1}{2-0} = \frac{x_2}{0+1} = \frac{x_3}{2-4}$$

(i,e) $\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2} = k$

(i,e) $x_1 = 2k$, $x_2 = k$, $x_3 = -2k$.

Hence the general eigen vector Corresponding to $\lambda=3$ is $X_2 = \begin{pmatrix} 2k \\ k \\ -2k \end{pmatrix}$

By putting $k=1$ we get the simplest

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Considering First Two Equations and applying rule of Cross Multiplication we have,

$$\begin{array}{ccccccc} x_1 & & & x_2 & & & x_3 \\ 2 & & 0 & & -1 & & 2 \\ -2 & & 1 & & 2 & & -2 \end{array}$$

$$\frac{x_1}{2-0} = \frac{x_2}{0+1} = \frac{x_3}{2-4}$$

(i,e) $\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2} = K$

(i,e) $x_1 = 2K$, $x_2 = K$, $x_3 = -2K$.

Hence the general eigen vector Corresponding to $\lambda = 3$ is $X_2 = \begin{pmatrix} 2K \\ K \\ -2K \end{pmatrix}$

By putting $K = 1$ we get the simplest

$$x_3 = \begin{pmatrix} 2k \\ -6k \\ 26k \end{pmatrix}$$

The simplest Eigen vector can be divided by taking $k = 1/2$

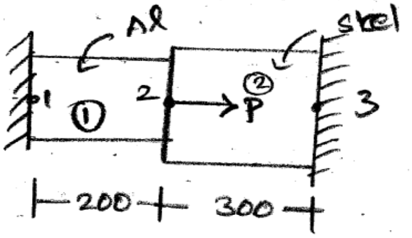
$$(i,e) \quad x_3 = \begin{pmatrix} 1 \\ -3 \\ 13 \end{pmatrix}$$

∴ The Eigen vector corresponding to $\lambda = 1$ is $(2,-1,-4)$,

∴ The Eigen vector corresponding to $\lambda = 3$ is $(2,1,-2)$,

∴ The Eigen vector corresponding to $\lambda = -4$ is $(1,-3,13)$.

— x —



G:- Al :

$A_1 = 1000 \text{ mm}^2$.

$E_1 = 0.7 \times 10^5 \text{ N/mm}^2$

$\alpha_1 = 23 \times 10^{-6}$ °c.

$P_1 = 4 \times 10^5 \text{ N}$

$T_1 = 30°$ c

$T_2 = 60°$C

Steel :- $A_2 = 1500 \text{ mm}^2$, $E_2 = 2 \times 10^5 \text{ N/mm}^2$

$\alpha_2 = 12 \times 10^{-6}$ °c.

To find :-

$k = ?$

$F = ?$, $u = ?$, $\sigma = ?$, $R = ?$

Sol :- Step 1 :

General Finite Element eqn,

$$\frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} = \begin{bmatrix} F \end{bmatrix},$$

For Ele(1)

$$\frac{A_1E_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\end{bmatrix}$$

$$\Rightarrow \frac{1000\times 0.7\times 10^5}{200}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\end{bmatrix}.$$

$$\Rightarrow 3.5\times 10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\end{bmatrix}$$

For Ele(2)

$$\frac{A_2E_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}$$

$$\rightarrow \frac{1500\times 2\times 10^5}{300}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}.$$

$$\Rightarrow 10\times 10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}$$

Ele(1) :-

$$1\times 10^5\begin{bmatrix}3.5 & -3.5\\ -3.5 & 3.5\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\end{bmatrix}$$

Ele (2) $\Rightarrow$

$$1\times 10^5\begin{bmatrix}10 & -10\\ -10 & 10\end{bmatrix}\begin{bmatrix}u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_2\\ F_3\end{bmatrix}.$$

Step 2:-

Global Stiffness Matrix:-

$$1\times 10^5\begin{bmatrix}3.5 & -3.5 & 0\\ -3.5 & 3.5+10 & -10\\ 0 & -10 & 10\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\\ F_3\end{bmatrix}.$$

$$1\times 10^5\begin{bmatrix}3.5 & -3.5 & 0\\ -3 & 13.5 & -10\\ 0 & -10 & 10\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}=\begin{bmatrix}F_1\\ F_2\\ F_3\end{bmatrix}$$

Step 3:

Force vector,

Ele (1):-

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = E_1 A_1 \alpha_1 \Delta T \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

$$= (0.7\times10^5 \times 1000 \times 23\times10^{-6} \times 30) \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(1×10^5)

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = 48300 \begin{bmatrix} -1 \\ 1 \end{bmatrix}. = (1\times10^5) \begin{bmatrix} -0.483 \\ 0.483 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} -0.483 \\ 0.483 \end{bmatrix}.$$

Ele (2):-

$$\begin{bmatrix} F_2 \\ F_3 \end{bmatrix} = E_2 A_2 \alpha_2 \Delta T \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= (2\times10^5 \times 1500 \times 12\times10^{-6} \times 30 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} F_2 \\ F_3 \end{bmatrix} = 1\times10^5 \begin{bmatrix} -1.080 \\ 1.080 \end{bmatrix}.$$

Step 4:- Axial Load is acting at Node (2),

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} -0.483 \\ -0.597 \\ 0.\text{[illegible]} \\ 1.080 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} -0.483 \\ 3.403 \\ 1.080 \end{bmatrix} \times 10^5.$$

W.K.T,

$$1\times10^{5}\begin{bmatrix} 3.5 & -3.5 & 0 \\ -3.5 & 13.5 & -10 \\ 0 & -10 & 10 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \underset{0}{=} \begin{bmatrix} 0.483 \\ 3.403 \\ 1.080 \end{bmatrix}$$

In fig, Nodes (1) and (3) is fixed, so, $u_1 = 0$, $u_3 = 0$.

$F_1 = 0.483$ $F_3 = 1.080$ $F_2 = 4.483$.

$13.5 \; u_2 = 4.483 \times 10^{5}$

$u_2 = 0.2520 \text{ mm}$

$u_1 = 0$ $u_2 = 0.2520$, $u_3 = 0$

Step 5 :- Stress (σ) :-

Ele (1) :-

$$\sigma_1 = \frac{E_1 \times (u_2 - u_1)}{l_1} - E_1 \alpha_1 \Delta T$$

$$= \left[\frac{0.7\times10^{5} \times (0.2520 - 0)}{200}\right] - 0.7\times10^{5} \times 23\times10^{-6} \times 30$$

$= 40 \text{ N/mm}^2$.

Ele (2) :-

$$\sigma_2 = \frac{E_2 \times (u_3 - u_2)}{l_2} - E_2 \alpha_2 \Delta T$$

$$= \frac{2\times10^{5} \times (-0.2520)}{300} - 2\times10^{5} \times 12\times10^{-6} \times 30$$

$= -240 \text{ N/mm}^2$.

Resultant (R):-

$$R = \{k\}\cdot\{u\} - \{F\}$$

$$= \begin{bmatrix}3.5 & -3.5 & 0\\ -3.5 & 13.5 & -10\\ 0 & -10 & 10\end{bmatrix}\begin{bmatrix}0\\ 0.2520\\ 0\end{bmatrix} - \begin{bmatrix}0.483\\ 3.403\\ 1.080\end{bmatrix}$$

$$= \begin{bmatrix}0-(0.882)+0\\ 0+\overset{3.402}{[illegible]}+0\\ 0-2.52+0\end{bmatrix} - \begin{bmatrix}0.483\\ 3.403\\ 1.080\end{bmatrix}\begin{matrix}3\\ \times10\end{matrix}$$

$$\begin{bmatrix}R_1\\ R_2\\ R_3\end{bmatrix} = \begin{bmatrix}-39.9\\ -[illegible]\\ -0.1\\ -[illegible]\\ -360\\ ~~3.6~~\end{bmatrix} = \begin{bmatrix}-39.9\\ -0.1\\ -360\end{bmatrix}$$

Verification:-

~~AF = RF~~

~~$4\times10^5 = \begin{bmatrix}-1.365\\ -0.001\\ -3.6\end{bmatrix}$~~

~~$4\times10^5 = -4.9$~~

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Applied Force $= F_1+F_2+F_3 = 0.483+3403+1.080 = 400$ N.

$R_1+R_2+R_3 = -39.9-0.1-360$

$= -400$ N

$\therefore$ AF = RF